E04039

THE REVOLUTION IN PHYSICS

First Photograph of the Track of a Positron.

The positron is produced by cosmic radiation. and while passing through a cloud chamber is deflected by a magnetic field. It passes through a ¼-inch lead plate (the horizontal band across centre of chamber), and thereby loses energy. On emerging it is travelling more slowly, and hence is more strongly deflected by the field. The photograph was taken by Carl D. Anderson (Physical Review, *March, 1933*).

Fig. 57 (see p. 199).

[*frontispiece*]

THE REVOLUTION IN PHYSICS

by

ERNST ZIMMER

with an introduction by

MAX PLANCK

translated, and with a preface, by

H. STAFFORD HATFIELD

London
FABER AND FABER LIMITED
24 Russell Square

FIRST PUBLISHED IN JANUARY MCMXXXVI
BY FABER & FABER LIMITED
24 RUSSELL SQUARE LONDON W.C. I
PRINTED IN GREAT BRITAIN AT
THE UNIVERSITY PRESS CAMBRIDGE

Introduction

The Author of this book has been known to me since his student days, and I am very pleased to be able to supply a few words by way of introduction—all the more because every serious and well-devised attempt to introduce wider circles to the recent progress of physics appears to me worthy of support. I hope that the present work will help to spread more widely, and found more firmly the view that the science of physics can demonstrate its contact with the life of humanity, not only by its technical and economic value, but also by the fact that it has much of importance to say in regard to our view of life as a whole.

MAX PLANCK

Translator's Preface

The widespread and growing interest aroused in the educated general public to-day by the most recent physical theories and discoveries is truly remarkable. These matters are by no means easy to grasp for those not possessed of a mathematical and physical training. However, their statement in non-mathematical language is becoming clearer and clearer as one expert after another sets himself to the task. The present volume is an important contribution to this process of clarification. Its object is not to acquaint the reader with technical details but to show him how it is that these new developments mean a complete change in man's conception of the external world.

The intelligent layman quite rightly believes that this changed conception, originating in mathematical and experimental researches which he can never hope to follow in detail, must necessarily be capable of statement in terms ultimately comprehensible to him. Some of our most eminent physicists, as well as philosophers of all schools, are feeling their way towards this statement. He is anxious to follow the formation of their opinions, for he knows that a new dominant attitude towards reality as a whole will ultimately emerge, which will profoundly influence human action, just as the attitude based upon classical physics has influenced it.

The rise of classical physics, four centuries ago, completely transformed man's view of the world in which he finds himself. Before the time of Galileo, Newton, and the other founders of the science, the world was ruled by angels and demons, after their time by immutable laws. Before their time, the world was confined in space and time; between Heaven above and Hell beneath, between the Creation and the Day of Judgment. After their time, the world was an insignificant globe of stone, travelling around the sun, itself a minor star among myriads scattered through infinite space. It had a history of millions of years of slow evolution from a

condition of glowing vapour; before it stretched a future also counted in millions of years, apart from the chance of its destruction by a cosmic catastrophe. And this history of the world, and of all upon it, was determined absolutely in every detail from the beginning of time. The same immutable physical laws governed the motions of the planets, and of the least atom forming part of them, and hence of our own bodies.

Man did not base upon this conception an attitude of complete fatalism, as would have been logical; quite the contrary. The practical conclusion was drawn that, since everything that happens in the world happens according to fixed laws, which man can discover, he can, by properly arranging objects in accordance with his knowledge, arrange for events to take place according to his desires. This conception, applied to physical and chemical technology, has borne the most marvellous fruits. The richer the fruits became, the more firmly rooted became the conception. Soon it was applied to biology, and to human affairs. But whereas the physicist proceeded step by step, setting up simple and fundamental laws which could be verified by experiments on simple systems designed to this end, 'thinkers' in other fields propounded 'laws' of the same absolute type, but necessarily unverifiable. Biological and social evolution, history, politics, psychology, were all studied with a view to the discovery of the forces acting, and the laws according to which they acted upon the objects studied. Finally, though none of these systems found that universal assent commanded by the experimentally verifiable, many of them found large bodies of adherents, and political and social thought and practice were greatly influenced by them. Indeed, at the present time, the belief that every human problem is to be treated as a technical problem, involving the construction upon a scientific basis of suitable machinery for its solution, is dominant.

Now the physicist tells us that he has penetrated behind and beyond this conception of the real world as a machine constructed in three-dimensional space out of interconnected parts, moving at a certain speed in a manner absolutely determined by the machine's construction. But he finds it very difficult indeed to tell us what conception will take its place.

Translator's Preface

The correct formulae describing physical phenomena are now known, but how we are to relate them to reality is at present a mystery. It required a Newton to devise a picture of reality fitting Kepler's and Galileo's discoveries. That picture is familiar enough to us now, but it must have seemed completely incomprehensible to the non-scientific when first it was put forward. However, it is not the attitude of the physicist and the philosopher, but that of thinking mankind at large, that is of decisive importance. It is for this reason that interest in these matters is so keen, and is growing. We are certainly assisting at the birth of a new era of human thought. We feel that it is to be an era of liberation, for we are very tired of the domination of the machine, in spite of the wonderful progress which it has inspired. For, ultimately, the classical picture of the universe is intolerable to man. It asks him to accept as the final reality a vast and sterile desert, in which the whole of his own history is an event as insignificant and momentary as the falling of a single drop of water upon the Sahara. This, we now know, is an appearance; reality cannot be described in terms of three-dimensional space and ever-flowing time. We are penetrating experimentally into its finest structure; we are finding perfect mathematical expression of our results. No machine-like picture can be constructed to fit these formulae, for they do not contain any mechanical elements. Therefore, the machine was an illusion; we have thrust our fingers through it, and found it unsubstantial. But how was the illusion produced?

H. STAFFORD HATFIELD

Contents

Contents

ACKNOWLEDGMENT

The following figures have been taken from other publications by kind permission of the publishers: Fig. 1, Newcomb-Engelmann, *Populäre Astronomie* (Wilh. Engelmann, Leipzig); Fig. 26 *a*, *b*, Rabinowitsch-Thilo, *Periodisches System* (Ferd. Enke, Stuttgart); Figs. 7, 22, 45, Haas, *Atomtheorie* (W. de Gruyter and Co., Berlin and Leipzig); Figs. 6 and 23, Heisenberg, *Die physikalischen Prinzipien der Quantentheorie* (S. Hirzel, Leipzig); Figs. 14 and 15, Grimsehl, *Lehrbuch der Physik* (B. G. Teubner, Leipzig and Berlin); Figs. 17 and 48, Kundt, *Vorlesungen über Experimentalphysik* (F. Vieweg und Sohn, Braunschweig); Fig. 19, Müller-Pouillet, *Lehrbuch der Physik und Mineralogie* (F. Vieweg und Sohn, Braunschweig); Figs. 43 and 44, Rupp, "Ueber Elektronenbeugung an dünnen Glimmerblättchen", *Die Naturwissenschaften*, 17. Jahrgang (J. Springer, Berlin); Figs. 54 *a* and *b*, London, "Die Bedeutung der Quantentheorie für die Chemie", *Die Naturwissenschaften*, 17. Jahrgang (J. Springer, Berlin).

THE REVOLUTION IN PHYSICS

CHAPTER I

The Nature of Matter in Classical Physics

The final goal of all natural science is to develop into mechanics.
—HERMANN VON HELMHOLTZ, 1821–1894.

We are going to consider our present-day picture of the physical world. We all of us carry around with us some kind of physical world picture, for we are in fact all of us physicists. Like the professional physicist, we all of us make use of our senses every day to observe what is going on around us. We see that white daylight is reflected by all objects in different colours, we hear the sound made by the November wind in the chimney, we feel the warmth of the air which surrounds us in springtime, and the cold blasts of winter, we smell the scent of the rose, we taste the bitter and salty taste of sea water.

These sense impressions are the source of all our knowledge of the outside world, and not only the physicist, but each one of us, works them over in a manner depending upon the nature of the human mind. We draw conclusions concerning cause and effect. If we see on a white tablecloth the varied colours of the rainbow, we trace them to the sun's rays broken up by the polished surfaces of a cut-glass vessel. Like the physicist, we believe that we are understanding the world around us when we think of it as filled with bodies each having a certain weight, each occupying a certain space; some may be moving through certain distances in certain times, that is to say, they may possess definite velocities. And just as does the physicist, we strive to set aside all subjective elements, to which the artistic world picture owes its charm, when we are endeavouring to understand the world as it exists independently of ourselves. And finally, all the concepts which we have formed in order to grasp the world, and

CHAPTER I

The Nature of Matter in Classical Physics

The final goal of all natural science is to develop into mechanics.
—HERMANN VON HELMHOLTZ, 1821–1894.

We are going to consider our present-day picture of the physical world. We all of us carry around with us some kind of physical world picture, for we are in fact all of us physicists. Like the professional physicist, we all of us make use of our senses every day to observe what is going on around us. We see that white daylight is reflected by all objects in different colours, we hear the sound made by the November wind in the chimney, we feel the warmth of the air which surrounds us in springtime, and the cold blasts of winter, we smell the scent of the rose, we taste the bitter and salty taste of sea water.

These sense impressions are the source of all our knowledge of the outside world, and not only the physicist, but each one of us, works them over in a manner depending upon the nature of the human mind. We draw conclusions concerning cause and effect. If we see on a white tablecloth the varied colours of the rainbow, we trace them to the sun's rays broken up by the polished surfaces of a cut-glass vessel. Like the physicist, we believe that we are understanding the world around us when we think of it as filled with bodies each having a certain weight, each occupying a certain space; some may be moving through certain distances in certain times, that is to say, they may possess definite velocities. And just as does the physicist, we strive to set aside all subjective elements, to which the artistic world picture owes its charm, when we are endeavouring to understand the world as it exists independently of ourselves. And finally, all the concepts which we have formed in order to grasp the world, and

the laws to which we suppose them to be subjected, are submitted by us, just as by the physicist, to the ultimate judgment of our sense impressions. We may settle a dispute on a controversial subject by the triumphant statement "but I saw it with my own eyes". And if we are known to be truthful, our opponent will give way, and recognise the finality of this appeal to experience.

The physicist proceeds in exactly the same way. In two respects only does he refine upon this procedure. In the first place, he performs experiments, that is to say, he does not merely observe nature, but puts questions to her, to which he receives replies. This requires creative imagination, for his experiments are usually planned to test an imaginary picture of the hidden workings of nature, and must therefore truly fit that picture. Secondly, these experimental experiences, which accumulate richly in the course of the centuries, are dealt with mathematically by the physicist. In this way their statement acquires an exactitude and certainty, which cannot be attained in any other way. The mathematical consequences of a physical theory are the acid test of its value. Many a scientific hypothesis, which has attained notoriety outside the ranks of physicists, fails to pass this test; for example the world-ice theory of Hörbiger. This gives, in a single bold sweep, a new and comprehensive picture of the cosmic process. But it cannot be upheld, for it fails to pass the test of calculation.

The physical picture of the world thus attained is transformed and refined continually under the pressure of experimental experience. "How has our physical world picture changed in the last twenty years?" asked the Berlin physicist Max Planck. "Each of us knows that the transformation which has occurred is one of the most profound which has ever taken place in the development of science." Before we consider this exciting transformation, we will first survey in a general way, in this and in the next chapter, the physical world picture as it appeared at the beginning of the century and shortly afterwards; the world picture of what is generally known to-day as "classical physics".

Physics began, apart from the experimental discoveries

made by Pythagorean philosophers concerning stringed instruments, with Galileo's experimental investigations on the free fall of bodies, made in the sixteenth century. The great Sir Isaac Newton then proceeded further along the same lines. He taught us to understand what appears to be the simplest of problems, the motion of bodies; his doctrine was so complete that, for two centuries, nothing of essential importance was added to it.

He said: every body is inert, that is to say, when the attempt is made to prove it, it opposes a resistance, its "mass", to motion; we will denote the number which specifies the amount of this mass with the letter m, and measure it in grams.* The number which denotes the velocity of a moving body or, as the physicist says, of a material point, we will call v, and measure it in centimetres per second, that is to say, by dividing the number of centimetres moved by the number of seconds required for the motion. When we multiply these two together, we form a new concept invented by Newton, the momentum or impulse $p = m \times v$. Instead of speaking of the velocity of a body, it is often preferable to speak of the velocity multiplied by the mass, that is to say, the impulse. The fundamental law of mechanics runs as follows: *the velocity of a moving body does not change of itself.* A ball which has once been given a push rolls over a smooth surface of ice without stopping; in order to stop it it is necessary to exert a "force". The measure of this force is, according to Newton, given by the rate at which it changes the impulse, that is to say, by the product of the mass of the body and the rate of change of its velocity. When no force acts, the impulse and therefore the velocity remain unchanged. This fundamental law is therefore called the *law of inertia*, or the *law of the conservation of impulse* or *momentum.*† If an elastic ball strikes full on a second ball of the same mass

* For scientific purposes the use of the metric system is practically universal, and it will therefore be retained in this book for the English reader. The English pound contains about 454 grams, the English yard about 91·45 centimetres. [Trans.]

† The terms *impulse* and *momentum* (Ger. *Impuls*, *Bewegungsgrösse*) are synonymous. We shall follow our author and use the first. [Trans.]

at rest, it loses the whole of its velocity and remains stationary, and the second ball now moves with the velocity of the first. It acquires the whole of the latter's impulse, the whole of its striking power. If the masses of the two balls are different, they rebound from one another so that the impulse acquired by the second ball is equal to that lost by the first. The total impulse remains unchanged.

This total impulse can only be changed by external forces. As Helmholtz and Robert Mayer found at the middle of last century, such external forces as a blow or a thrust can convey to the ball what is called "energy". The ball is able by virtue of this energy to move faster than before; for instance, it is now able to run far up a slope. As it does so, its velocity and hence its energy of motion will gradually diminish, and at a certain height it will come to rest for a moment. We then say that it possesses energy of position. When it rolls down again, this energy of position is again transformed into energy of motion. A moving railway train may also lose energy of motion as a result of the application of the brakes; the brake shoes then become hot. The energy of motion has been transformed into heat energy. The steam engine shows us that it is also possible to produce energy of motion from heat energy; when a steam engine drives a dynamo, the heat energy is transformed into electrical energy. This is a form of energy which is very easily transformed; thus in the vacuum cleaner or in the tramcar it is transformed into energy of motion, in electric irons and stoves into heat, in the electric lamp into light or radiant energy. In coal, radiant energy has been stored in the form of chemical energy, as a result of the action of the sun upon the primeval forest in earlier ages.

What is true of impulse is also true of energy. It can neither be created nor annihilated. It can only be conveyed from one body to another or from one form to another. This law is known as that of the *conservation of energy*. Newtonian physics is, however, unable to tell us how great is the quantity of energy contained in a given piece of matter.

Newton taught us a second matter of great importance, namely, a new mathematical method of following the motion of a "material point" in time. It has the following result:

if at a definite moment I know exactly the position and impulse of a body, that is to say, the magnitude and direction of its velocity at that moment, I know what we call its "state". If now I also know all forces which subsequently act upon the body from outside, the method invented by Newton allows me to calculate its "state" at the next moment of time, and the next, and so on through all future time. All these steps may be summed up, and the whole history of the body may be stated mathematically. This mathematical method is known as the "calculus of infinitesimals", or the "differential calculus".

The power of this method of Newton's is extraordinary. If, for example, we know at a given instant the position and velocity of the planets, as determined by the astronomer, it is possible to calculate their paths for all future time and also for all past time, if only we know how great are the attractive forces acting between each pair of them. Newton in his law of gravitation stated how this attractive force may be calculated from the masses of the two heavenly bodies and their distance apart. Everyone must be impressed when he learns how exactly it is possible to calculate the time of a future or past eclipse of the moon or sun.

Newton and the whole of classical physics regard the law of causality as absolute; that is to say, the world of matter is regarded as free from all chance, and everything that happens in it is conditioned by strict necessity. Not a speck of dust could have any other position than it actually has without the whole cosmos being in some way influenced for the whole of eternity afterwards. The perfection, the power, and the fertility of Newtonian mechanics produced permanent effect. We know how great was its influence upon Kant's theory of knowledge. This deterministic view of nature celebrated its greatest triumph in the discovery of the planet Neptune, which is on the confines of the solar system and a very feeble luminary. Leverrier, sitting at his desk, calculated its existence and position in the year 1845. He suspected that the small irregularities exhibited by the farthest planet then known, Uranus, could only be caused by a still more distant body, and calculated the position in the sky of this disturbing

individual. The astronomer Galle then actually found it almost precisely in the calculated position (Fig. 1). In the same manner, a still more distant planet, Pluto, was discovered in the year 1930.

Laplace, a great French mathematician and physicist, who lived at the end of the eighteenth century, said: "a Spirit who knew at a given moment all the forces existing in nature, and the relative position of all existing things or elements composing it, would, if he were able to submit all these data to mathematical analysis, be able to comprehend in a single formula the motion of the greatest heavenly body and of the lightest atom; nothing would be uncertain for him, and future as well as past would lie open before his eyes." This omniscience of Laplace's World Spirit, or Demon, is a vivid statement of the strict necessity postulated by the law of causality as applied by classical physics to world events; and even if we do not agree with the materialists—using the word in the epistemological sense—that all happening in the world, including the mental and spiritual, is to be explained by the motion of tiny particles, we shall certainly agree with the physicists of the past, and with all non-physicists, that necessity and law rule all that happens within the realm of physics. But Laplace's Demon involves the truth of an assumption which will probably be regarded as self-evident, namely that it is possible, at least in principle, to determine the position of a body with any required degree of exactitude. In actual fact, no one had the slightest doubt on this point during the classical period of physics, nor until a few years ago.

When Newtonian mechanics had been worked out, the laws governing the motion of bodies appeared to be completely known. Every airship and aeroplane which flies in safety over our heads, convinces us in the most satisfying way that the physicist is perfectly aware of the laws according to which bodies move. When, therefore, Einstein asserted, in the year 1905, that we were far from such certainty, and that all our mechanics is only true for motion at relatively low speeds, the greatest astonishment was manifested. He taught that at velocities comparable with the velocity of light, which is the fastest motion known to us

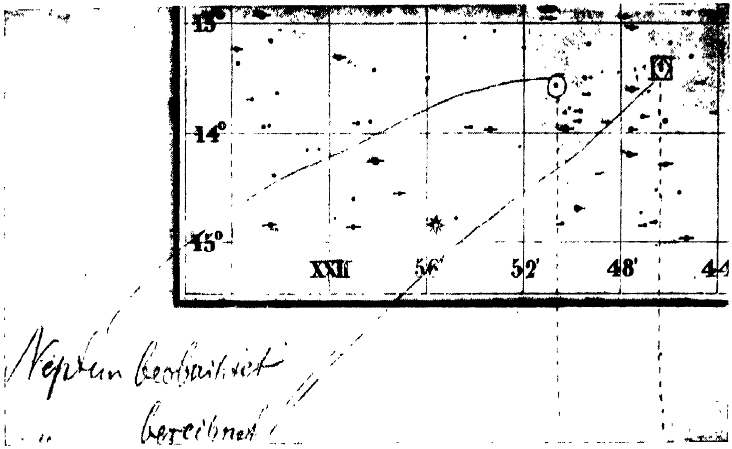


Fig. 1. Stellar map showing observed and calculated position of Neptune when discovered.

[*to face page* 6]

(300,000 kilometres per second) a large departure from the Newtonian laws takes place; we shall shortly become acquainted with bodies which move at such speeds. Physicists experienced a strong sensation of emancipation, emancipation from prejudices hitherto noticed by no one, concerning the measurement of space and time.

We will not attempt at this point a short survey of the relativity theory. It is too difficult for such an attempt. We will only select, from the whole complex of questions of which it treats, one of its most important results, and describe this result shortly; we will then discuss two important consequences which we shall require later.

The mechanics of Newton and his successors had failed at one point, namely, the explanation of the cause of that mysterious and instantaneously operative attractive force which exists between all heavenly bodies. Here the relativity theory is successful, but only at the cost of assuming that space is ruled, not by Euclid's familiar geometry, but by a different, non-Euclidean geometry. What is the meaning of this? Our clearest view is perhaps obtained by means of a picture which has been used again and again. Let us imagine two beings, one living upon a limitless flat table-top, the other upon a sphere. This also is without boundaries for the being confined to its surface. The sphere we will suppose to be very large compared with our imaginary being, so that he does not notice its curvature. Both of them study geometry. The straight lines of the dweller on the sphere are the great circles on the latter, as for example, a meridian, or the equator. Both beings, we suppose, are able to draw triangles and measure their angles. The dweller in the flat world finds the sum of the angles of a triangle to be equal to 180°, while the dweller in the curved world finds them to be always greater than 180°. This is easily seen as regards a triangle made up of two arcs of meridian from the pole to the equator and a part of the equator; in addition to the two right angles between the meridians and the equator, making up 180°, we have the angle between the meridians at the pole. The dweller on the table-top can draw a line parallel to any straight line through a point outside it, and this parallel will

not cut the straight line however far it is produced. The dweller on the sphere is unacquainted with parallel "straight" lines, for great circles always cut one another. He, therefore, has a different, non-Euclidean geometry.

We know that this is a result of the curvature of the sphere-dweller's two-dimensional world. If, therefore, we are going to study non-Euclidean geometry in our three-dimensional world, a geometry analogous to the spherical geometry just considered, we have to imagine this space as curved. We are unable to do this. Is the notion therefore nonsensical? Certainly not; because we are not able to form a clear picture of anything, we are not necessarily dealing with something meaningless. We are not able to imagine this curved three-dimensional world, but we are able to calculate its properties. Just as our Euclidean mathematical theory of solid bodies is nothing more than an extension of our two-dimensional plane geometry into three dimensions, so have mathematicians extended the generalised geometry of two-dimensional spherical surfaces into three, and indeed more, dimensions. If therefore we imagine the beings we have been discussing to be flat, that is to say, to possess only two dimensions, and therefore to have no conception of the existence of a third dimension, we can form by analogy a picture of our own position in the three-dimensional world. Our flat sphere dweller could know nothing of the curvature of his space. Both of them live in worlds which are two-dimensional, and they know nothing of a third dimension; but while the world of the table-top is limitless in extent, the other is limited, it returns upon itself. In the same way universal space is, for Einstein, limited in extent, although very very large. The question of what is beyond the Universe* if it is after all limited in extent is meaningless, for the world has no boundaries. Our sphere-dweller also can never reach a boundary, although his sphere is limited in size; for he cannot get out of his two dimensions.

* The word "world" is now, chiefly under German influence, displacing the word "universe" as meaning the whole of objective physical reality. In fact, the word "universe" is now often used to denote a single nebular system. [Trans.]

Such is the world of the general theory of relativity. It is curved everywhere like a sphere, but where there are masses, for example stars, the curvature is increased as if there were humps on the sphere. And gravitation? There is no such thing; it is simulated by the humps in curvature which occur near masses. Instead, we have the law of the conservation of impulse with its consequence that, in the absence of such a force as gravitation, every body moves in the straightest possible path. In the neighbourhood of masses, therefore, a body is constrained to move, as it were, in grooves around the hump. Now the mathematical calculation, which is extremely difficult, shows that such a body will move exactly as if it were under the influence of a force of the same amount, or nearly the same amount, as the Newtonian force of gravitation. The motion is slightly different; according to Einstein, the elliptical paths taken by the planets around the sun must themselves slowly turn about the latter. This slight effect must be strongest in the case of planets close to the sun, Mercury being the closest. Its path should turn through 43 seconds of arc in one hundred years (Fig. 2). This fact, and its correct magnitude, had long been known to astronomers, and could not be explained by the Newtonian theory of gravitation.

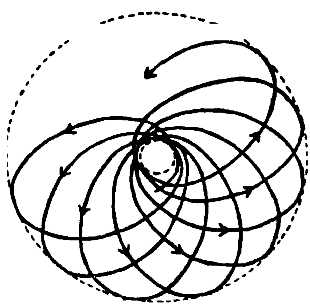


Fig. 2. Rotating ellipse (from Sommerfeld).

The reader may perhaps find all this rather uncanny or fantastic; and, in reality, the matter is even more difficult since all we have said is not true for the ordinary three-dimensional world to which we are accustomed, but for a four-dimensional, imagined world which is obtained by adding to the three dimensions of space (left—right, forward—backward, up—down), time (past—future) as a fourth dimension. We are accustomed to the fact that statements which we make about one of these three dimensions of space, only have a "relative" validity. It depends upon my point

of view whether I judge an object to be to my left or my right, and a person facing me is of the opposite opinion. To say that a thing is left or right of another is to make a relative statement. "Absolute" truths valid for everyone, the laws of nature namely, must not contain words such as left and right, and must not refer only to one of the three dimensions of space. The theory of relativity teaches us correspondingly that the true laws of nature do not refer to single dimensions of the four, and therefore not to the world as evident to our senses in space and time, but to a four-dimensional ideal world at the back of it, a world which is non-Euclidean in construction. At any rate, these laws can only be simply expressed as regards such a world. The phenomena of nature which we perceive with our senses are projections in space and time from that ideal world. This relativity of our concepts of space and time has not been forced upon us by philosophical considerations, but by physical experiments. The physicist regards the theory of relativity, which Planck calls "an impressive structure of wonderful harmony and beauty" as the crown of classical physics.

Nevertheless, we may suppose that many readers will be glad to learn that we shall hardly need to make use of the aforesaid notion of the theory of relativity. But two other consequences of the theory will be of some importance to us, namely the following. Firstly: mass is not unchangeable. If I compare the mass of a lorry with the mass of a light car I say that the mass of the lorry is greater because a greater force is needed to accelerate it at a certain rate than is required for the light car. This mass, which we measure in kilograms or pounds, we usually regard as unalterable in the case of any given body. In the relativity theory this is not the case. At very high velocities, the mass of a body actually increases on account of its higher speed. A kilogram then becomes more than a kilogram, and it requires a greater force to produce the same increase in its velocity. The body resists increase in its speed more and more as that speed increases, and this resistance finally becomes enormous, with the result that velocities greater than the velocity of light are impossible. According to the relativity theory, light does not merely hold

the record for speed; its velocity is the greatest velocity which can possibly be possessed by any moving body. If anything could be found to move faster than it, the theory of relativity would be disproved.

A further curious result is the following. This optical magnitude, the velocity of light, determines something else quite different, namely the value of the total energy contained in a body. We may recollect that Newton's theory could not tell us this amount. According to the theory of relativity, the energy E of a body at rest, if its mass is m grams, is given by

$$E = m \times c^2;$$

here c denotes the velocity of light, expressed in centimetres per second, that is to say, $30{,}000{,}000{,}000 = 3 \times 10^{10}$. Therefore $c^2 = c \times c = 9 \times 10^{20}$, an enormously large number.* A very small mass thus represents a very large energy and, on the other hand, the concentration of a great deal of energy results only in a very small mass. The changes in energy which we are able to produce by means of our technical methods, are minute as compared with the enormous amount which would be at our disposal if we were able to dematerialise the mass of a brick and turn it into energy. Nernst once compared the world to an island made of gun-cotton, and we can only be glad that the physicist has not yet found a match which would set it alight. But something of the kind may possibly happen in the middle of the hottest fixed stars. We can calculate how great is the amount of energy radiated to us in the form of light by the sun. What is the source of this energy? According to the law of conservation of energy it can only be derived from other energy. All other sources of energy are too small. But if matter itself is nothing more than a form of energy, the sun might convert small portions of its own substance into large quantities of radiant energy according to the equation $E = m \times c^2$. The matter forming the

* Scientists have a convenient way of expressing large numbers. 5000 is 5×1000, and 1000 is $10 \times 10 \times 10$, or 10^3. Hence 5000 may be written 5×10^3 or $5 \cdot 10^3$. Any elementary algebra will show us that $\frac{1}{1000}$ is 10^{-3}, hence $\frac{5}{1000}$, or 0·005, may be written 5×10^{-3}. Again

$$1000 \times \tfrac{1}{1000} = 10^3 \times 10^{-3} = 10^{3+(-3)} = 10^0 = 1. \text{ [Trans.]}$$

heavenly bodies may thus transform itself into light energy, and light may perhaps be retransformed into matter. This would be the most general form of the law of conservation of energy. We shall learn more about this later on.

Radiant energy or light will be discussed in the next chapter. For the present we are dealing with matter. It is characterised by the two quantities impulse and energy.

So far we have only spoken of motion in the great world visible to us. But there is another world, a small one, smaller than the microscope can reveal to us, the world of atoms and molecules. We will call the former the macro-world, and the latter the micro-world. The most fundamental notions concerning the micro-world were already developed by the Greek philosophers. The philosopher Democritus is regarded as the founder of the conception of atoms, and as the first scientific materialist. He was a genuine seeker after truth, and a man of high ideals. He is said to have exclaimed: "I would rather discover a single connection of cause and effect than become King of the Persians." His search for a single and universal explanation of the varied and manifold phenomena of existence led him to deny the truth of the everyday assumption that the material substance of all bodies is continuous. He rather regarded all matter as built up of single, indivisible, very minute bodies, which he called atoms. Its structure is grained, like a mass of peas. These atoms are devoid of all qualities such as odour or colour; their sole properties are the possession of a certain weight, and the occupation of a certain volume.

The atomic theory has been taken over by modern science from Greek philosophy, and we shall shortly consider its rebirth and development in modern times. Another notion of ours also originated with the Greeks. This is the assumption that "elements" exist, namely, fundamental substances which cannot be transformed one into the other, but which, by combining together in innumerable ways, yield all known compound substances. Chemists only accepted this idea after long investigation. Scarcely any other idea has, in the course of centuries, been subjected to so much investigation in the hope that it might prove to be false. This investigation was

one of the chief occupations of the Alchemists. Their plans were daring in the extreme. They aimed at producing an artificial man, at discovering the Philosopher's Stone, which was to bring all possible good fortune to the earth, and finally, at obtaining gold from ignoble metals. There was much admixture of deception and greed in this serious search for truth. No one has ever seen the Philosopher's Stone, nor yet the "Homunculus";* and though the alchemists succeeded in making many important chemical discoveries, they were never able to make gold from lead. Neither of these chemical elements can be transformed into the other. The positive result of their attempts at this transformation was to prove the existence of such fundamental elements which resist all attempts to transform or decompose them by chemical methods.

Round about the year 1800, the founders of modern chemistry were led by the results of their investigation concerning the formation of chemical compounds, to combine the two hypotheses, namely, the existence of unchangeable elements, and the grained structure of matter. They once again maintained that matter is to be regarded as built up of very minute structural units, the atoms. As many different kinds of these exist as there are different unchangeable fundamental substances, such as iron, lead, gold, sulphur, chlorine, phosphorus, hydrogen, oxygen, etc. Elements differ from one another as regards the weight of their atoms. The atoms then combine together in smaller or larger groups to form the smallest units, the molecules, of which chemical compounds, such as water, salt, sugar, alcohol, are made up. Bodies as we know and see them are made up of millions of these small units or molecules.

For a long time scientists for the most part refused to regard atoms and molecules as anything more than purely imaginary notions. They refused to admit that a piece of solid matter, such as the paper on which this is printed, might be made up of individual atoms separated from one another by empty space. Only a few decades ago many leading scientists were still denying the real existence of atoms. To-day

* Rechristened "Robot" in our times. [Trans.]

no physicist or chemist has any more doubt of their reality than of the reality of visible bodies. If they are real, then a gram of any given element must consist of a certain very large number of atoms. Numerous experiments can be devised which lead to the possibility of deducing this number from their results. Practically the same number is found from experiments of very different types, and it is this fact which is regarded as conclusive evidence of the reality of atoms. The weight of an atom of hydrogen is found to be $1{\cdot}649 \times 10^{-24}$ grams; an ounce is thus just about as many times heavier than a hydrogen atom, as it is lighter than the earth.

All this belongs to classical physics, and all we shall do is to remind the reader of a few more facts with which he is probably familiar. The *atomic weight* of an element represents (roughly) the ratio of the weight of one of its atoms to the weight of a hydrogen atom. Fig. 3 shows all the chemical elements with their atomic weights, arranged according to the "periodic system", of which we shall have much to say later on. The number of atoms with which we ordinarily have to do is, naturally, correspondingly large. In one cubic centimetre of a gas at standard barometric pressure and at ice temperature, there are twenty-seven *trillions* of molecules. The diameter of an atom is about one hundred-millionth of a centimetre. The portions of matter which we can handle and see therefore contain an enormous number of atoms, and the properties of matter known to us in our daily life are those of a very large number of molecules.

The actual existence of atoms and molecules having been proven, it became the task of the physicist to explain the whole properties of the infinitely varied world of matter as resulting from the weight and motion of these minute particles; the materialist had in addition to explain also the phenomena of mental life. The assumption was made that they moved according to the same laws as the bodies of normal size to which we are accustomed. However, science was far from being in the happy position of Laplace's Spirit, which was able to calculate the movement of an immense number of bodies in all directions. But it was just the immensity of

their number which enabled physicists to proceed as does the director of an insurance company. It is true that we do not know *who* are the people destined to be killed by accident in London next year, but the insurance expert is always able to predict fairly accurately how great their numbers will be. When we are dealing with a large number of individuals, and when the number of atoms and molecules which take part in a physical or chemical process is very much greater than the number of people who pass along a busy London street in the course of a year, we are able to dispense with a knowledge of the fate of individual particles, and to rely upon calculations of probability. The motions of individual atoms are still to be regarded as causally determined, for any other supposition appears for the moment impossible; the path of every single atom is set for it unalterably for all future time when its initial "state" and the forces acting upon it are determined. But the physicist cannot follow the motion of a single atom, and is obliged to be content with stating the average motion of a very large number. This method, however, leads to the understanding of a large number of phenomena.

Its greatest triumph has been to bring all the known facts comprising the science of heat under the aegis of mechanics; this required a further assumption, the truth and value of which have been proved by its success. This assumption is as follows: atoms and molecules are in continual motion, this motion being faster the higher the temperature. Heat energy is thus identified with the energy of motion known to us in mechanics. At higher temperatures molecules move irregularly in all directions like snowflakes in a blizzard, and we have matter in the gaseous form. The pressure exerted by a gas upon the walls of a vessel, for instance by steam upon the walls of a boiler, is caused by the continual volley of blows from the flying molecules. The power of these blows can become so great as to drive forward the piston in the cylinder of a steam-engine; it may even become strong enough to destroy the walls of the boiler and produce an explosion. The molecules of escaping steam, when no longer subjected to resistance, spread freely into space on account of

Period	I	II	III	IV	V
K	1 Hydrogen H = 1·008				
L	3 Lithium Li = 6·94	4 Beryllium Be = 9·02	5 Boron B = 10·82	6 Carbon C = 12·00	7 Nitrogen N = 14·01
M	11 Sodium Na = 23·00	12 Magnesium Mg = 24·32	13 Aluminium Al = 26·97	14 Silicon Si = 28·06	15 Phosphorus P = 31·02
N	19 Potassium K = 39·10 29 Copper Cu = 63·57	20 Calcium Ca = 40·08 30 Zinc Zn = 65·38	21 Scandium Sc = 45·10 31 Gallium Ga = 69·72	22 Titanium Ti = 47·90 32 Germanium Ge = 72·60	23 Vanadium V = 50·95 33 Arsenic As = 74·93
O	37 Rubidium Rb = 85·44 47 Silver Ag = 107·88	38 Strontium Sr = 87·63 48 Cadmium Cd = 112·41	39 Yttrium Y = 88·92 49 Indium In = 114·8	40 Zirconium Zr = 91·22 50 Tin Sn = 118·70	41 Niobium Nb = 93·3 51 Antimony Sb = 121·76
P	55 Caesium Cs = 132·81 79 Gold Au = 197·2	56 Barium Ba = 137·36 80 Mercury Hg = 200·61	57 Lanthanum La = 138·92 58–71 Rare earths 81 Thallium Tl = 204·39	72 Hafnium Hf = 178·6 82 Lead Pb = 207·22	73 Tantalum Ta = 181·4 83 Bismuth Bi = 209·00
Q	87 ?	88 Radium Ra = 225·97	89 Actinium Ac = 228	90 Thorium Th = 232·12	91 Protactinium Pa = 234?

58 Cerium Ce = 140·13	59 Praseodymium Pr = 140·92	60 Neodymium Nd = 144·27	61 Illinium Il = 146?
65 Terbium Tb = 159·2	66 Dysprosium Dy = 162·46	67 Holmium Ho = 163·5	68 Erbium Er = 167·64

Fig. 3. The Periodic

VI	VII	VIII	0	Period
			2 Helium He=4·00	K
8 Oxygen O=16·00	9 Fluorine F=19·00		10 Neon Ne=20·18	L
16 Sulphur S=32·06	17 Chlorine Cl=35·46		18 Argon Ar=39·94	M
24 Chromium Cr=52·01	25 Manganese Mn=54·93	26 Iron Fe=55·84 27 Cobalt Co=58·94 28 Nickel Ni=58·69		N
34 Selenium Se=78·96	35 Bromine Br=79·92		36 Krypton Kr=83·7	
42 Molybdenum Mo=96·0	43 Masurium Ma=98?	44 Ruthenium Ru=101·7 45 Rhodium Rh=102·91 46 Palladium Pd=106·7		O
52 Tellurium Te=127·59	53 Iodine I=126·92		54 Xenon X=131·3	
74 Tungsten W=184·0	75 Rhenium Re=186·31	76 Osmium Os=190·8 77 Iridium Ir=193·1 78 Platinum Pt=195·23		P
84 Polonium Po=210·0	85 ?		86 Radon Rn (Em)=222	
92 Uranium U=238·14				Q

62 Samarium Sm=150·43	63 Europium Eu=152·0	64 Gadolinium Gd=157·3
69 Thulium Tu=169·4	70 Ytterbium Yb=173·5	71 Lutecium Lu=175·0

Alternative Names

Beryllium *or* Glucinum
Hafnium *or* Celtium
Illinium *or* Florentium
Niobium *or* Columbium
Radon *or* Niton
Lutecium *or* Cassiopeium

System of the Elements.

their rapid motion. When the steam is cooled down, the velocity of the molecules diminishes, and they approach one another more and more closely, until they are like human beings in a crowd, nearly touching. We then have matter in the liquid state. Further cooling results in lessening their motion still more, until each molecule only moves to and fro about a fixed position. They behave like a large number of soldiers drawn up in formation, each executing movements without changing his position (Fig. 4). We see that this arrangement of atoms and molecules in fixed positions, of course in three dimensions, may be made according to many patterns. It is in this way that we explain the large number of crystalline forms assumed by solid bodies.

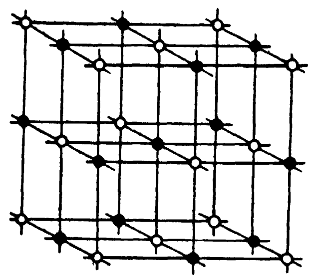


Fig. 4. Face-centred cubic lattice of common salt, built up of sodium and chlorine atoms.

This theory of the solid, liquid, and gaseous forms of bodies made up of atoms and molecules is called the "kinetic theory". It leads at once to the notion of a lowest temperature, an "absolute zero", at which all motion of the particles ceases. It has long ago been known and calculated that this temperature is very near to $-273°$ C., but it is only quite recently that experimenters have come within a fraction of a degree of it. The reality of this point is sufficiently proved by the enormous increase in the difficulty of approaching nearer to it, the nearer we already are. The last fraction of a degree is costing almost as much labour as the previous 270°. But great progress has been made recently, particularly by Dutch physicists. If anyone should ever succeed in obtaining a lower temperature, the whole kinetic theory would be thrown open to doubt. So far it has stood the experimental test as well as the theory of relativity.

It was this picture of the material world as constituted of moving atoms which was worked out in detail in the nineteenth century. The atoms themselves were thought of as the smallest stones or bricks out of which matter is made up. Physicists and chemists mostly regarded them as minute

indivisible spheres, without worrying about the question whether one can really imagine such indivisible small bodies to exist. They left all speculation of this kind to philosophers. As far as physics was concerned, experiment always showed that atoms were indivisible.

The aspect of affairs was changed when the investigation of the passage of electricity through liquids and gases began in the last third of the past century. The electric current can be sent through a vessel of water, made conducting by the addition of a little sulphuric acid, by connecting the poles of a battery to pieces of sheet metal dipping in the liquid. Let us suppose, for example, that we connect the positive pole of a battery to a sheet of copper, and the negative pole to a sheet of platinum (Fig. 5). We then find that not only does the current pass through the liquid, but that the copper also passes from the positive to the negative pole. At first the platinum, at which the current leaves the liquid, is silver white, and the liquid is colourless; but very soon the platinum becomes red with transported copper, and the liquid becomes, not copper red, but blue. It now contains electrically charged copper atoms, copper "ions" as they are called, and these are different in nature from ordinary copper atoms, and of a different colour. Similar results are obtained with other metals such as silver, nickel, and so on. These metallic atoms also travel with the electric current as ions from one pole to the other. If we weigh the amount of metal transferred, and calculate from this result the number of atoms transported, at the same time determining the amount of electricity which has flowed, we arrive at a curious result. We are forced to regard electricity, as well as matter, as divided into minute equal portions, one of which travels with each atom of silver, while two travel with an atom of copper, three with one of aluminium, and four with one of tin. We never find fractions of these amounts. We are therefore obliged to assume that indivisible atoms of electricity also exist.

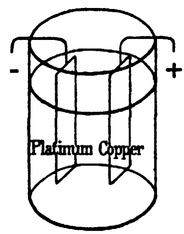


Fig. 5. Passage of electric current through acidulated water.

The magnitude of this elementary electrical charge, the atom of electricity, is extraordinarily small. It amounts to $1{\cdot}591 \times 10^{-20}$ units, when we take as a unit the amount of electricity which flows in ten seconds when the current is one ampere. Classical physics regards all electric currents as streams of such "electrons".

Electrons are also known in a free state. When lightning flashes between two clouds, or sparks pass between the conductors of an electrical machine, the high electric voltage causes electrons to fly through the air. The atoms and molecules of the air, when struck by them, become luminous. This effect of the electrons is much better seen when the air through which they are passed is reduced in pressure. The brilliant colours first discovered by Geissler, which result when electric currents are sent through gases under low pressure contained in glass tubes, have in recent times become one of the commonest sights in our streets at night. They result from the action of electrons which, driven by the high electric potential, strike the atoms of the gas and cause it to emit its characteristic light. We are all now familiar with the characteristic red colour of neon, the commonest gas used in signs.

When we reduce the pressure of the gas sufficiently, as in an X-ray tube, these effects disappear, and we are left only with an activity which proceeds from the negative pole of the tube, the so-called "cathode". This active material does not proceed towards the positive plate, which may be anywhere in the tube, but at right angles to the surface of the cathode in a straight line. When it strikes the walls of the tube it causes them to become luminous. Other substances also shine in characteristic colours when they are struck by these "cathode rays".

What are cathode rays? If they are generated in an exhausted tube containing a tiny windmill, so placed as to be struck by them, the vanes of the mill turn round as if they were being struck by tiny projectiles. These projectiles move in straight lines. The motion of the windmill shows that they give up their speed, that is to say their impulse, to its vanes. And, like true moving particles, they also possess energy. If

they are concentrated upon a small piece of platinum, all their energy is transferred to it, and it becomes warm. But these flying particles have another property: if they are allowed to pass through a slit and then to strike a sensitised screen which they cause to become luminous, we find that they are diverted by bringing a magnet near them. They move in a circular path, like a stone which is attached to a string and whirled round. The string exerts a continued even pull at right angles to the direction of its motion. The physicist knows that electrically charged bodies travelling at a high speed through the field of a magnet are also acted upon by a steady pull at right angles to their direction of motion. From the direction in which they are diverted he is able to deduce that they are negatively charged, a fact which is already extremely probable, since they are thrown off by the cathode, the negative pole of the tube. Cathode rays are therefore negatively charged corpuscles.

What is the amount of their charge and their mass; that is to say, what are they made of? The theory of electricity teaches us that particles, carrying a charge e—usually measured in ampere-seconds or coulombs—and travelling along a tube to which the electric voltage V—usually measured in volts—is applied, acquire electric energy amounting to $e \times V$. This energy they then possess in the form of kinetic energy. But this may also be expressed as depending on their mass m and their velocity v. The voltage of the cathode ray tube is known. The values of e, m, and v, are unknown, and we have therefore to discover them. Experiments in which the energy of the cathode rays is measured by transforming it into heat, or in which their deflection by known magnetic and electric fields is measured, enable us to set up sufficient simultaneous equations to calculate e, m, and v.

The result of all these experiments and calculations is as follows. The velocity of the particles varies greatly, according to the voltage applied to the tube, but is very large indeed, between one-tenth and one-third of the velocity of light with the high voltages usually employed. Even a few volts are sufficient to give them velocities of some hundreds of miles a

second. On the other hand, the charge and mass always turn out to be the same, quite independently of the applied voltage, and of the material of which, the electrodes, the tube, or the gas contained in it consist. The cathode rays always represent the same electrically charged substance, always possessing the charge of an atom of electricity as given above, and the minute mass of 9×10^{-28} grams. This is 1800 times lighter than the lightest atom known to us, namely hydrogen. It is therefore no substance already known to us, but pure electricity. These minute corpuscles are called "electrons", and they have been found to be contained in all matter. Whatever the material constituting the electrode from which they proceed, they themselves are always identical. The electrons therefore form a part of the atoms of all elements, and we are no longer able to consider atoms as indivisible.

In what form are the electrons contained in the atom? By means of a very important discovery, we have been able to come to more definite conclusions on this point. This was the discovery of radioactivity by Pierre and Marie Curie in the year 1894. Certain elements, possessing the highest known atomic weight, were found, in contrast to our experience of all other elements, to decompose of themselves; that is to say, to change more or less rapidly into others having smaller atomic weight. In this process they emit "rays" of three different kinds, which have been designated by the Greek letters α (alpha), β (beta) and γ (gamma). The gamma rays will be dealt with in the next chapter, the beta rays have been shown by their deflection in a magnetic field to consist of very fast electrons, and thus to be identical with cathode rays as already known to us in the cathode ray tube. The alpha rays are deflected by a magnetic field in the opposite direction to the beta rays; they are particles carrying a positive electric charge, but are not electricity separated from matter, as are electrons. Negative electrons are easily obtained free in many different ways, all of which involve splitting them off from atoms. When an electrically neutral atom is thus robbed of one or more of its negative electrons, it is left as a positively charged "ion". Quite recently

"positrons", that is, particles of electricity of the same mass as an electron, but positively charged, have been observed. Their existence is fleeting, for reasons which we shall discuss later. Alpha rays (or particles) are atoms of helium, which have lost two electrons; the element helium appears as element number 2 in the periodic system and is the gas used in America in place of hydrogen for filling airships. The alpha particles, when emitted by the decomposing atoms of radioactive elements, move with enormous velocities, although not so fast as the beta particles.

Very many radioactive elements are known, which can be arranged in series, in such a way that one is transformed into the next. They all emit alpha or beta rays of varying velocities. A characteristic of each element is its "half-period", that is to say, the time required for half of the atoms of any given portion of it to disappear by conversion into the next element of the series. In the case of radium this period is 1730 years. From this we can calculate that in a second one atom out of 37,000,000,000 is transformed. It is furthermore remarkable that the half-periods of the various elements differ enormously. The shortest lived is radium C′, which already decomposes to the extent of one-half in a ten-millionth of a second; the longest lived is uranium I, which requires 5,000,000,000 years for the same thing to happen. Radium A has a half-period of three minutes. Classical physics can tell us nothing about the way in which the decay of the radioactive elements, and the great differences in its rate, are to be explained. Finally, these elements change, after a whole series of transformations, into common elements such as lead, bismuth, thallium, which are not radioactive.

The paths taken by the alpha and beta rays through the air can be rendered visible (Fig. 6). The method was invented by C. T. R. Wilson, and consists in sending them into a "cloud chamber", that is to say, a vessel filled with air somewhat over-saturated with water vapour. The alpha particles ionise the air as they pass through it, and each ion attracts to itself water molecules, which condense to form a drop. The track of the particle is thus shown by a white cloud which can be photographed. The alpha particles capture on

their way first one electron, and then another, when they become electrically neutral and are no longer able to cause drops to condense. The result is that their tracks end quite suddenly.

The English physicist Rutherford was led by a consideration of these photographs to draw some very important conclusions, founded upon previous suggestions of the German physicist Lenard. The alpha rays in the photographs pass through air; when Rutherford and Chadwick sent them through metals and other bodies, they obtained similar paths, usually straight, but sometimes exhibiting slight, and occasionally very sharp, turns. When alpha rays shoot through the spaces between the atoms of the matter through which they are passing, they are entirely unaffected, on account of their enormous speed; but sometimes they strike an atom and force their way into its interior. Even then, the lighter constituents of the atom, the electrons, are quite incompetent to deflect the paths of particles which are 7000 times heavier than themselves. Very powerful forces indeed must be at work to deflect these minute projectiles while travelling at such high speeds; nevertheless, they are forced far out of their paths, as we see by the sharp bends in the photographs. Rutherford imagines the whole mass of the atom to be concentrated in what he calls a nucleus, which carries a positive charge, which balances the negative charge of the outer electrons. From the deflection of the alpha rays he is able to calculate the charge and the size of these nuclei. They are very small indeed, only one ten-billionth of a centimetre. Since the whole atom, as we know, has a diameter of only one hundred-millionth of a centimetre, the nucleus occupies only one hundred-thousandth part of the diameter of the atom: which is about the same ratio as the size of a cathedral to the whole earth. The whole atom is thus empty, or rather contains only a few electrons, themselves extremely small and held in position by the powerful electric field of the nucleus.

When we calculate the amount of this charge we get a very curious result. We find that the charge on the nucleus of any atom is proportional to the number which states its order in the periodic system of the elements. The hydrogen atom must

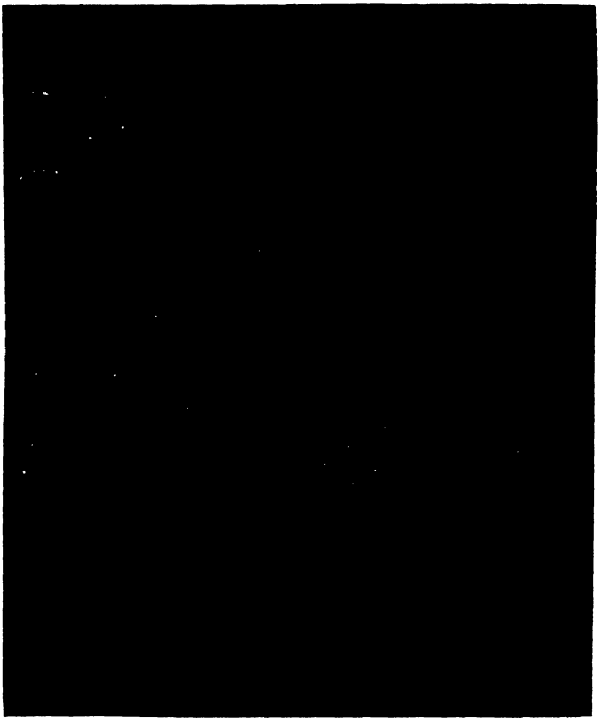


Fig. 6. Cloud tracks of alpha rays.

[*to face page 24*]

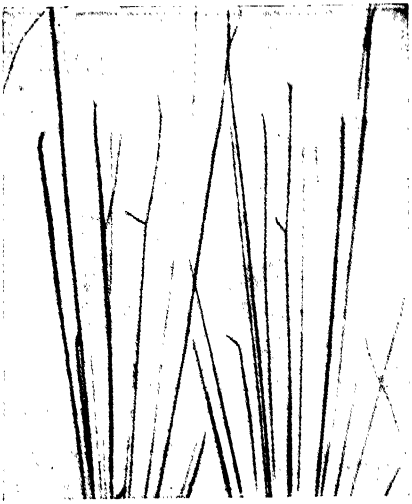


Fig. 7a. Passage of alpha particles through hydrogen (Blackett).

Fig. 7b. Disintegration of nitrogen nuclei by alpha particles (Blackett).

[*to face page 25*]

therefore be supposed to be made up of the two fundamental constituents of matter, a nucleus carrying a charge of one unit, and called the *proton*, which may be compared in size to a house situated at the centre of the earth, and further on the surface of the atom (and by comparison, the earth) an electron, possessing an equal and opposite charge to that of the nucleus. The nucleus of the helium atom has a double positive charge; it must consist of four protons (hence its atomic weight 4), and two electrons, the result being a charge $4-2=2$. Two further electrons revolve about this nucleus. As we proceed from element to element, the positive charge of the nucleus and the number of the external electrons increase each time by one unit, until we reach the heaviest element uranium. Its atomic weight 239, and its atomic number 92, show that 92 electrons surround a nucleus consisting of 239 protons, to which we must add $239-92=147$ nuclear electrons, in order that the whole atom may be electrically neutral. All atoms and thus all matter appear, unless experiment shows otherwise, to be made up of two different kinds of corpuscles of equal and opposite electric charge, but differing in weight, the positively charged proton being 1800 times heavier than the electron.

It is impossible in this work to present the enormous mass of experimental evidence upon which these remarkable conclusions are based, but we bear these results in mind. In the centre of a cloud of electrons, forming the atom, and unchangeable by anything that the chemist and the physicist can do, we have this minute world in itself which we call the nucleus. All that we know is that this nucleus is itself a complicated structure, and that the nuclei of the heaviest elements are for some reason or other unstable, and sooner or later break up; when they do so they send out electrons or beta particles, travelling at very high speeds.

For a long time, no means were found by which the breaking up of atomic nuclei could be influenced or brought about artificially; but in 1919 Rutherford succeeded in disintegrating atoms artificially. We can readily understand that this could only be effected by means of projectiles having enormous energy concentrated in very small space; such pro-

jectiles are given us in the alpha rays. By their means, Rutherford and his collaborators succeeded in realising, to a certain extent, the ancient dream of the alchemist whose hope was to transform lead into gold. It is true that Rutherford did not actually change lead into gold, but only nitrogen into hydrogen, and then only in excessively minute amount. Fig. 7 *a* shows what happens when an alpha ray hits a hydrogen nucleus full on. The ray is itself strongly deflected from its original direction, while the hydrogen nucleus acquires a part of the impulse and travels on nearly in the direction in which it was hit. The hydrogen nuclei travel much further on account of their light weight: their "range" is quite definite. When Rutherford fired alpha particles through nitrogen gas, he obtained particles possessing the same range as those in the experiment with hydrogen. We must conclude that hydrogen nuclei were shot out of the nitrogen atoms. In such an experiment, only one alpha particle out of 45,000 hits a nitrogen nucleus and causes a proton to be liberated. Fig. 7 *b* shows two such tracks. Blackett's experiment in 1925 showed that nitrogen, when it is thus bombarded with alpha rays, that is to say helium nuclei, is transformed into oxygen. Later, Rutherford and his colleagues, as well as Kirsch and Pettersson in Vienna, have succeeded in shooting protons out of many other nuclei by means of alpha rays. These disintegration experiments may be regarded as the first beginnings of "nuclear chemistry".

CHAPTER II

The Nature of Light according to Classical Physics

About twenty-five years ago, when the sciences once again had reached a high peak....

GOTTFRIED KELLER, Das Sinngedicht.

Not many subjects of physical investigation possess for everybody such an immediate attraction as light. In the classical period of their science, that is to say from the seventeenth century to the nineteenth, physicists paid the closest attention to it. As their knowledge of its properties grew, their insight into its nature increased. The fundamental ideas of this classical science of optics are widely known even to non-physicists, and much of this chapter will not be new ground to most readers. But it is absolutely necessary first to make at least a general survey of the interesting development of the science of light, and of its concepts, if we are to understand the extraordinary revolution which has taken place in the present century in our ideas concerning matter.

To begin with, we must avoid a misunderstanding which has caused a great deal of unnecessary controversy. It is familiar to those who have interested themselves in the nature of our perception of colour, and was at the bottom of the bitter controversy on light between Newton and Goethe. The word *light* is used to designate two quite different things, firstly that physical something external to ourselves, which is emitted by a burning candle, a lamp, or a star, is reflected by mirrors, and is refracted by lenses; and secondly, the sensation which that something causes in our eyes. When the physicist asks what light is, he is not seeking to know about the physiological processes taking place in our eyes, but about that reality which exists outside ourselves. It is of this alone

that we shall speak, for in physics we are not engaged in learning something about ourselves but, as far as that is possible, only about the external world.

If we seek to know something about the nature of a person or a flower, we need to study them long and closely. The same is true of this subtle thing called light, and we shall need to collect every kind of experience concerning it in order to gain some insight into its nature.

The results of such collected experience led, at the end of the seventeenth century, to various hypotheses being formed to explain its properties. The most obvious one is that of which the chief defender was Isaac Newton. It assumes that light consists, just as does matter, of minute particles, which are projected by a source of light in straight lines in all directions with a speed of 300,000 kilometres per second. This "corpuscular" theory is first of all able to explain all phenomena connected with the rectilinear propagation of light, for example, the formation of sharp shadows.

But we are acquainted with other properties of light. When it strikes bodies which are not transparent, it is reflected. Every flat mirror exhibits the fundamental law of all reflection. A ray of light which strikes a mirror at a certain angle glances off from the surface at the same angle: the angle of reflection is equal to the angle of incidence. All other reflection by curved mirrors, such as the concave shaving mirror, the silvered glass ball, the distorting mirrors in fun fairs, or the parabolic mirrors used in searchlights, all these cases are only consequences of this law, as we find on closer examination. The corpuscular theory gives us an adequate explanation of it. We only need to think of the billiard ball which glances off from the cushion according to exactly the same law.

When we examine the passage of light through a transparent body, we meet with a new effect, refraction. A coin resting upon the bottom of a vessel containing water appears to our eyes to be much higher than it is in reality (Fig. 8). The rays of light proceeding from any point *A* of the object are bent as they leave the water and enter the much less dense air, and they appear to an eye situated at *B* to come, not from

A, but from a higher point *C*. For the same reason, a stick dipping into water appears to be sharply bent upwards at the point where it enters the water. These facts of the refraction of light were easily explained by Newton on the assumption that the molecules of the transparent body exert attractive forces on the light corpuscles. As the light passes through air or water, the attractive forces act equally in all directions, and hence cannot divert it from a straight path, but as it passes the boundary from the less dense air to the denser water, the attraction of the molecules pulls it downward (Fig. 9).

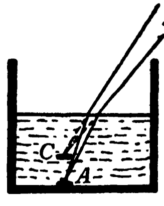


Fig. 8. Refraction of light: a coin under water.

Now let us turn to another important and well-known effect. If we send white light through a prism, that is to say, a piece of glass or other transparent material of which the sides are not parallel (the lustres on Victorian chandeliers and vases were such prisms) we find that the white light proves to be,

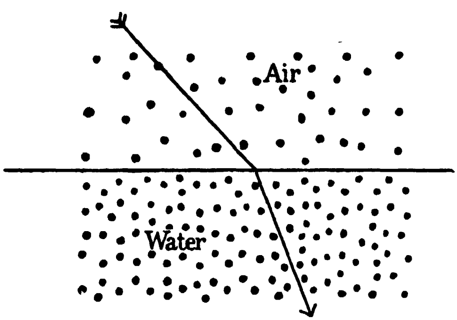


Fig. 9. Refraction of light according to the corpuscular theory.

not the pure and simple thing we take it to be on the evidence of our eyes, but a mixture of all the colours of the rainbow; the prism separates them by refracting them to a different degree. These colours of the *spectrum*, as the physicists call the whole range, are in order: red, orange, yellow, green, blue, indigo, and violet. The red is the least refrangible, the violet most. This can also be explained on Newton's theory, by

ascribing to the light corpuscles a different mass according to their refrangibility; the violet corpuscles would have the least mass, and the red the greatest.

These phenomena suffice to enable us to understand the chief optical instruments, such as the microscope, the projector, and the camera.

In spite of the fact that the corpuscular theory was able to explain all properties of light known at that time, the great Dutch physicist, Huygens, nevertheless propounded an entirely different view. He regarded light as a wave process, analogous to that with which we are familiar in sound. When a tuning-fork giving a note A on the piano is struck, we know that it vibrates to and fro, 435 times a second. We also know that it does not send out minute corpuscles, or anything of a material nature, but is the source of a state of vibration

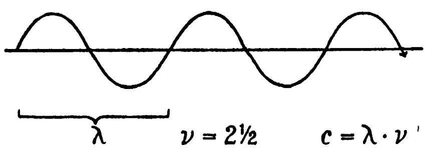


Fig. 10. Wave.

which travels through the air to our ears in the form of waves, just as waves travel over the surface of water. According to Huygens, we are to think of light as similar in nature.

Let us examine the model of such a wave (Fig. 10). Imagine a rope fixed to a hook in the wall by one end. We take hold of the other end, stretch the rope not too tightly, and shake the end we are holding rapidly and regularly. Waves then run to and fro along the rope; each wave has a crest and a trough. We characterise a corpuscle by the amount of its energy and its impulse. A wave is characterised by its wave-length, that is to say, the distance between two crests, and by its frequency, that is to say, the figure which states how many times per second the end of the rope was shaken up and down; this figure also tells us how many waves travel in a second along the rope. The physicist usually designates the wave-length by the Greek letter λ (lambda) and the frequency by the Greek ν (nu). If we look at the figure,

we see that, if $\nu = 2\frac{1}{2}$ waves pass a given point per second, and the length of each is $\lambda = 2$, the rate at which they travel, c, will be equal to $\lambda\nu = 5$, since the crest of a wave appears to move through a distance 2 in $\frac{1}{2\frac{1}{2}}$ seconds.

The properties of waves are best known to us in the case of waves travelling on the surface of water. If a stone is thrown into a still pool of water, we see a wave-front travel out from it at a definite speed in all directions. The wave motion thus appears to travel in straight lines, and the wave theory is thus able to explain the rectilinear propagation of light.

How does the matter stand as regards reflection? This is best seen when the wash of a fast moving boat travelling over smooth water meets the shore; the waves are seen to be reflected from it at the same angle as that at which they strike it. The wave theory thus also leads to the law that the angle of reflection is equal to the angle of incidence.

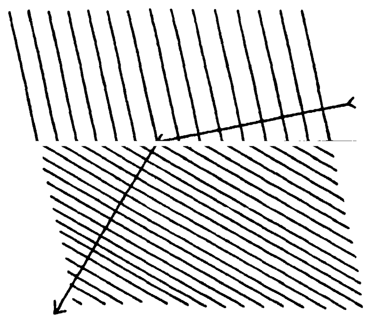


Fig. 11. Refraction of plane waves.

The law of refraction can also be shown to agree with the wave theory. We can observe the proof of it when waves travelling in deep water, arrive at a place where the water becomes shallower (Fig. 11). The decomposition of white light into colours can be quite simply explained by assuming that each colour of the spectrum has a different wave-length, that of red being the longest, and that of violet being the shortest.

Waves exhibit a new property which is not immediately obvious. If plane waves travel through a large opening, they proceed unchanged beyond it also as plane waves. But when the opening is made smaller and smaller its edges produce a disturbance; the wave front is bent round at its edges, and the wave is "diffracted". When the opening is very much reduced in size, waves no longer travel straight at all. Instead,

the small opening seems to form the starting point of a new wave which travels uniformly in all directions (Figs. 12, 13).

As a consequence of this, light, if it is a wave motion, should not pass straight on through a narrow slit, but should be diffracted to both sides. We are already accustomed to something of this kind in the case of sound waves. We can hear a noise in the next room through a door slightly ajar, not merely when the opening in the door is exactly between us and the source of sound, but when we are in any part of the room. The corpuscular theory would exclude this kind of thing altogether, for particles can only move in straight lines. This is also true of our ordinary experience of light. We hear

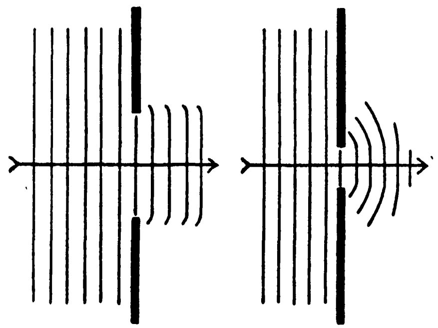


Fig. 12. Passage of a plane wave through a wide, and a narrower, opening.

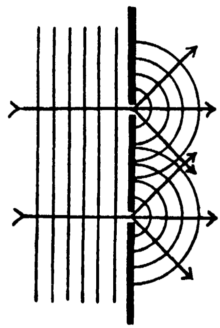


Fig. 13. Passage of a plane wave through two very narrow openings.

the horn of a motor car coming around the corner, but we do not see it: the sound waves are bent but not the light waves. However, the wave theory is not so easily disposed of; it may be that the waves of light are so short that their diffraction can only be observed when they pass through exceedingly fine slits. Let us therefore take the case when light passes through a very fine slit, or better still, a series of very fine slits close together; what is called a "grating". The light waves should then pass through the slits, spread out behind them, as in Fig. 13, and overlap one another. We now have to consider what will happen when several waves meet.

Let us first consider two such waves meeting one another. In the case of waves on the surface of water in a harbour, we

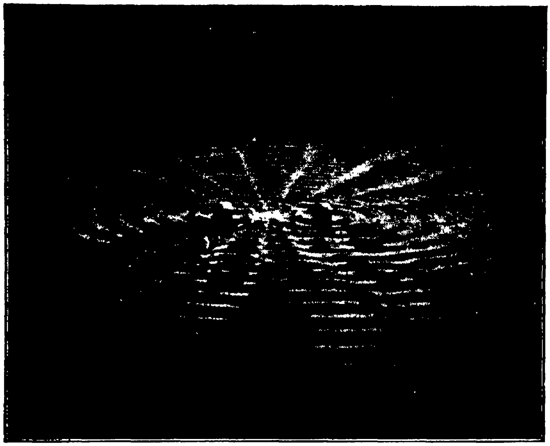


Fig. 14.

Fig. 15.

Interference of two water waves (Grimsehl).

[*to face page* 33]

can see what happens when the waves wash against the smooth surface of the quay. The crests of the waves can become much higher, for at every point where two crests come together they will produce a higher crest, and where two troughs come together, a deeper trough: we then have waves with crests twice as high and troughs twice as deep. But we also find places where a wave trough and a wave crest come together; the result will then be mutual destruction, and no wave at all. Figs. 14 and 15 show this meeting, or "interference" of two waves. It is easy to recognise the points where the crests and the troughs are enlarged. At such a point the waves have travelled distances from their starting-point which are different from one another by one or several whole wave-lengths. The points of no wave in between are those where one wave has travelled a distance exactly one-half wave-length, or a multiple of this, further than the other.

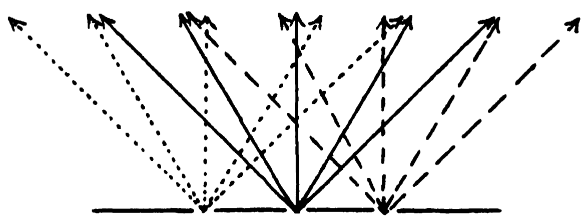


Fig. 16 a. *Waves spreading out from grating.*

The same thing must happen when waves pass through a grating. In this case also the waves, when they meet behind the grating, will alternately strengthen and weaken one another. If light is a wave motion, we ought to be able to observe something of this kind happening. Two rays of light when they meet will not necessarily always give rise to increased illumination: when the crest of a wave meets the trough of another wave, we have the case that light added to light gives darkness. An interference experiment of this kind must be regarded as decisive in favour of a wave theory, for according to a corpuscular theory, the addition of two sets of corpuscles together should only give an increased effect, not a diminished one.

This experiment in the interference of light can actually

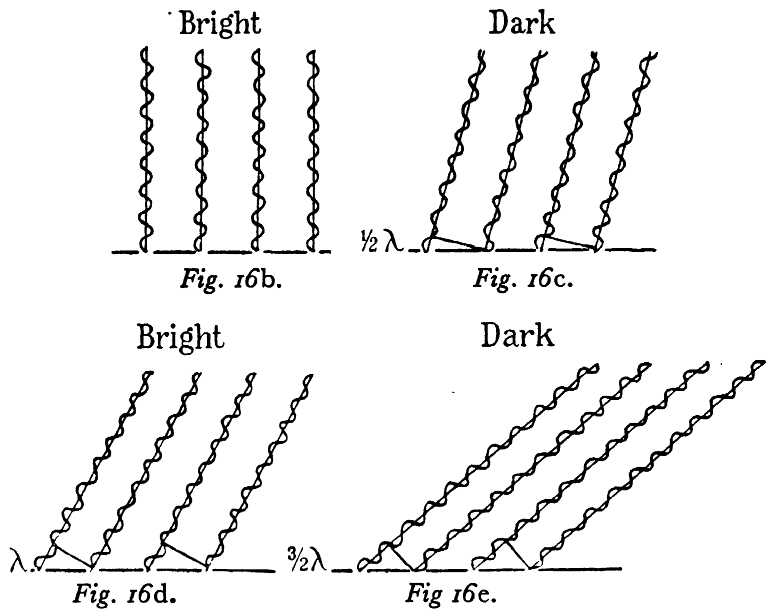
be carried out. On account of its importance, we will discuss it a little more fully. Huygens propounded his theory before it had been performed, and had to meet the objection that nothing resembling interference was then known in the case of light. He had to take refuge in the assertion that it would be necessary to use an extremely fine grating, since the waves of light are much shorter than those of water. Hence anything like a garden fence is of no use. But means were soon devised for ruling on a plate of glass many hundreds of fine parallel grooves per inch, whereby we have transparent strips and opaque strips alternating, and very close together.

Let us suppose that such a grating is set parallel to a flat white screen, and that a parallel beam of light is sent through the grating at right angles to it, so as to fall on the screen. If the corpuscular theory is correct, we should expect to see upon the screen a series of bright lines separated by dark spaces, the distance apart of these being equal to that of the lines ruled on the grating—in other words, a shadow of the rulings on the grating would appear on the screen. (Since these are very close together, we should, of course, not be able to distinguish them at all by the naked eye.)

Now consider what happens according to the wave theory. From each point between the rulings of the grating, waves spread out in all directions (Fig. 16*a*, p. 33). The waves, we must remember, are all vibrating together as they strike the grating. Hence waves arriving from neighbouring openings in the grating, at a point on the screen in the track of the original light, are still vibrating together (Fig. 16 *b*), assuming the screen to be so far from the grating that all these rays of light are parallel. Now take a point on the screen to the side of the track of the light (Fig. 16*c*). Here we will suppose that each opening in the grating, going from left to right, is just half a wave-length nearer to the point on the screen we are considering. Then the waves from each adjacent opening will be vibrating in opposite senses when they meet the screen; the trough of one will coincide with the crest of the next, and we shall have a dark line on the screen. Farther to the right we can find a point where the path of adjacent waves differs by a whole wave-length, and here the waves all reinforce one

another and we get a bright line (Fig. 16*d*). Farther along still (Fig. 16*e*) we have a difference of path of one and a half wave-lengths, and here again crest meets trough, and we get darkness.

If we use pure red light, for example, we get exactly the effect expected (Fig. 17). The figure shows that light actually does turn round a corner in the most unexpected manner, but in full agreement with the wave theory. Each diffraction



Waves passing through a grating. Alternate strengthening and weakening of vibration.

image is produced by all waves travelling from the grating towards it strengthening one another's effect. The wave theory of Huygens allows this effect to be predicted, while it is quite incomprehensible from the point of view of the corpuscular theory. Two rays of light can actually result in darkness, when wave crest just meets wave trough. We are thus obliged to conclude that the corpuscular is wrong and the wave theory is right. Light does not consist of corpuscles, but of waves.

With blue light exactly the same thing happens (Fig. 17 II),

but the distances apart of the images are smaller. We must now consider why this should be. It will be necessary in this case to deal in actual figures, for this will give us a deeper insight into the matter and will be of advantage later on. Consider Fig. 18. Here we have the grating with a very small spacing a between each groove. On the screen (AB) we have

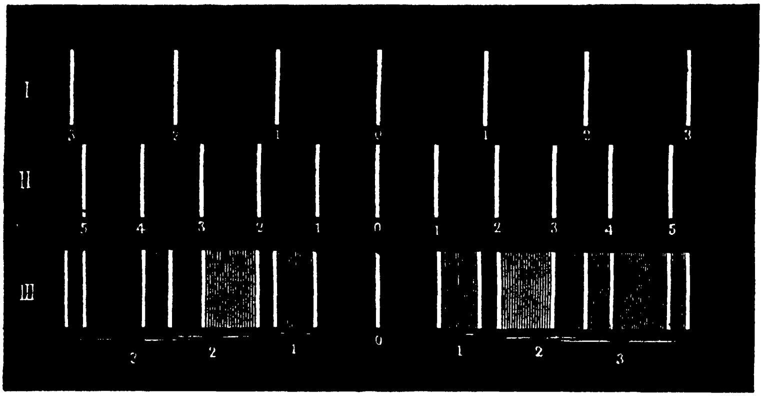


Fig. 17.

I. Diffraction of red light by a grating. All the diffraction images are red.

II. Diffraction of blue light by the same grating. All images are coloured blue. Their distances apart are only about half as great.

III. Diffraction of white light by the same grating. The middle line is white. On each side we have a series of wide diffraction images, each of which contains all colours of the spectrum from red to violet. We should obtain these by combining I and II, and adding the intervening colours. The diffraction spectra of the second, third, and higher orders partially cover one another.

a distance d between the two first bright lines, while the much longer distance between grating and screen is denoted by c.

The two triangles are similar, hence $b:c=\lambda:a$. The difference between the values of b in the case of blue and red light arises from the fact that the blue light has a shorter wave-length λ than the red light; hence, since c and a are the same, b must be smaller. The spectrum colours lying between red and violet will also have wave-lengths lying between these two extremes. When the grating is illuminated with white

light, each separate diffraction image must give the whole spectrum; these spectra partially overlap. Only the line in the middle will be white, since at this point all wave-lengths arrive in such a way as mutually to increase their effects. This is actually found from experiment, as we see from Fig. 17 III. Any of us can easily make an experiment of this kind, for we all possess in our eyelashes a suitable grating. We must look at a candle some yards away from us in darkness with our eyes half closed; we then see numerous images of the candle flame on both sides of it. All of them, with the exception of the one in the middle, are in rainbow colours, blue always being on the side towards the candle.

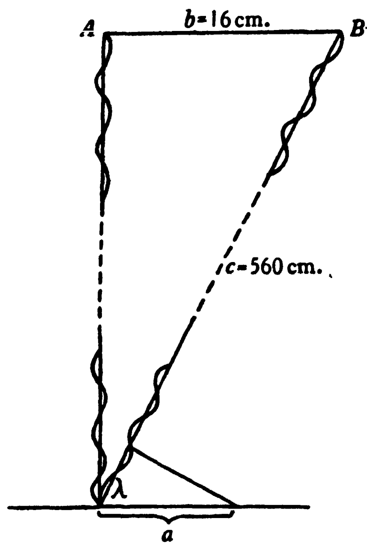


Fig. 18. Diffraction of waves by a grating; a = *spacing of grating,* b = *distance between two bright lines,* c = *oblique distance between grating and screen.*

A grating can be used to determine the wave-length of any kind of light. If, say, we take for the measurement of red light a grating having 400 lines to the centimetre, then $a = 1/400$ cm. If we have a screen at a distance $c = 560$ cm. the distance apart of the images, or "fringes" as they are technically termed, $b = 16$ cm. Our equation can be written in the form

$$\lambda = \frac{ab}{c},$$

and $\lambda = \frac{1}{400} \times \frac{16}{560}$ cm. $= \frac{1}{14{,}000} = 0{\cdot}00007 = 7 \times 10^{-5}$ cm.

For the extreme violet, we get half this result, namely $3{\cdot}8 \times 10^{-5}$ cm. The wave-length of visible light is thus exceedingly small. If we combine these values with the known velocity of light $c = 3 \times 10^{10}$ cm. per second, we can use the

formula $c = \lambda\nu$ to calculate the frequency of vibration ν of light of different colours. Thus we get for red light

$$\nu = \frac{3 \times 10^{10}}{7 \times 10^{-5}} = \frac{3}{7} \times 10^{15} = \text{about } 4 \times 10^{14},$$

that is to say about 400,000,000,000,000 vibrations per second. Since the wave-length of violet light is half as great, its frequency is twice as great. By analogy with the musical scale, we can say that violet light is an octave higher than red light, for a musical note is raised an octave in pitch by doubling its frequency.

What are we to expect if we attempt to use a coarser grating? From the equation $\lambda : a = b : c$, we can deduce the following. If we have a grating in which a is much larger, the fraction $\lambda : a$ is much smaller, and $b : c$ is therefore also much smaller, and when the distance c is the same, b will be much smaller, so that the single images will come much nearer together; finally, when the spacing is very wide, they will cover one another, and we shall get a single strip of white upon the screen. By a wide spacing, we do not mean anything like a garden fence, but simply a space large in comparison with the wave-length. Since the latter is so small, tenths of millimetres are already large. We therefore only notice diffraction phenomena when light passes through very fine slits; it appears to travel through wide slits in straight lines, as if it consisted of corpuscles. The corpuscular theory thus gives correct results so long as we are not working on a scale comparable with the wave-length of light. On this scale, the theory fails us. Light is certainly a wave motion, for diffraction experiments prove it.

The converse case is also of importance, namely when the spacing of the grating used becomes extremely small. The smaller a is, the farther apart do the "fringes" become, and finally, when $a = \lambda$, the first fringe formed obliquely, is formed in a direction at right angles to the ray of light, and hence never reaches the screen however wide we make it. In other words, obstacles in the path of the light no longer affect it when they are as large as, not to say smaller than, its wave-length. They have absolutely no effect on the path of the

ray. We have the direct image formed, and the diffraction images on either side of it disappear. In the same way, water waves such as come in shore from the sea, are not influenced by an obstacle such as a post, the breadth of which is shorter than their wave-length. We cannot therefore use light to see objects which are smaller than its wave-length. Now we know that the atoms of matter are about 10^{-8} cm. in diameter, that is to say, a thousand times smaller than the wave-length of light; hence we cannot see them. Light passes through the air without change in straight lines, for the molecules have no more effect upon it than a post upon the waves of the sea. If we are dealing with minute bodies somewhat larger than the wave-length of light, we cannot even then obtain sharp images of them, such that we are able to recognise their form; we get enlarged diffraction images which lie far to the side of the path of the light. If a ray of bright sunshine enters a room, we see the tiny particles of dust in the air, which we are otherwise unable to see; or rather, we see the diffraction images formed in our eyes to the side of the path of the light.

If instead of a single grating, we take two, and set them so that their lines cross, we have a grating with a new set of interference phenomena, which can also be calculated by means of the wave theory of light (Fig. 19). If we hold very finely woven material, and look at a candle through it, we are able to see very fine examples of such diffraction figures. These figures change according to the angle at which the lines of the gratings cross one another; and we can thus conclude from the nature of the diffraction images, the arrangement and spacing of the different points of the grating.

Let us leave this for the present. There are many other interesting observations to be made with spectra. The most important is the following. Light exists which we are unable to see. The spectrum formed by the light from a white-hot body by means of a prism or a grating does not extend merely from red to violet, but in both directions beyond this range. There is "ultra-violet" light beyond the violet and having shorter wave-lengths and there is "infra-red" light beyond the red with longer wave-length than it. Our eyes are

insensitive to both these kinds of light, and yet they exist and are truly light. I will endeavour to prove this fact.

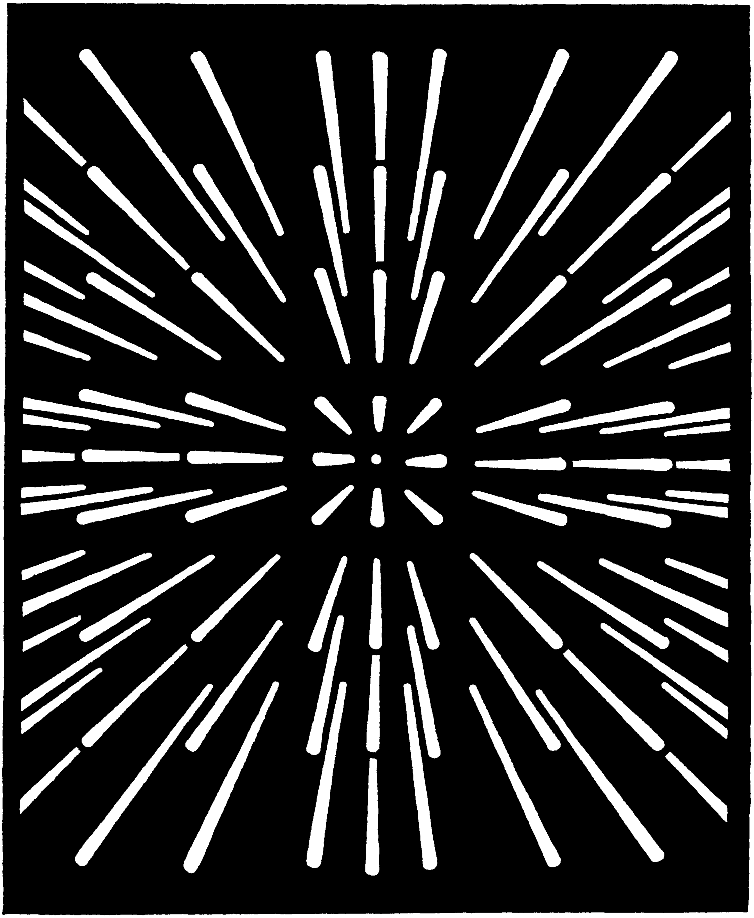


Fig. 19. Diffraction picture of a white point source formed by two crossed gratings. The point in the middle is white. All stripes are coloured with the spectral colours; each being red—yellow—green—blue from outward end inwards.

The existence of ultra-violet light can easily be shown by means of a piece of paper coated with what is called a fluorescent substance. Such a substance possesses the power

of changing the wave-length of light which falls upon it, and reflecting it, therefore, in a changed colour. Numerous such substances exist, for example fluorescein. A paper screen coated with this dye reflects violet light as blue-green. It is a curious fact that in every case where this remarkable change of colour is produced, the change is always in the direction of lowering, and not of raising, the frequency of the light. This is known as Stokes's law; we must postpone the explanation of it until the next chapter. For the moment, the important fact for us is, if we throw a spectrum on this fluorescent screen, we find that the screen shines blue-green beyond the limit of the violet end of the spectrum, although the eye is unable to perceive the ultra-violet light which is falling upon the screen, but only that of longer wave-length which it is giving out. This is a proof that the spectrum contains light of higher frequency than the violet, and this is transformed by the fluorescein into blue-green light of lower frequency. It has thus been possible to investigate ultra-violet light for many octaves beyond the visible spectrum. These invisible rays differ in certain respects from the visible rays. For example, they have a different action upon the human body, and this is made use of in medicine as "artificial sunshine"; furthermore, they have a powerful action on the photographic plate.

Light waves of less frequency than the red, the infra-red as they are called, can be detected by their own peculiar properties, the most important of which is their heating effect. By means of sensitive thermometers we can show that the heating effect already perceptible in the yellow and red part of the spectrum, is much stronger in the infra-red. These heat waves can be also followed through many octaves, the frequency decreasing, and the wave-length increasing, as we get farther away from visible light. Such waves of length greater than a third of a millimetre can be detected.

These invisible kinds of light are true light, although we cannot see them, for they have been proved to follow all the laws of visible light: reflection, refraction, interference, and diffraction. It so happens that our eye is not tuned to more than the single octave in which visible light is comprised.

Fig. 20 shows the whole spectrum so arranged that the wave-length doubles from division to division of the scale, and the frequency falls each time by one-half.

After classical physics had assured itself by such means as we have described, of the wave nature of light, and had also discovered the enormous range covered by these waves, it tackled no less successfully another question of equal importance, that of their nature. What is it that actually vibrates billions of times a second? Huygens imagined these vibrations to be something like sound in nature. A tuning fork, the string of an instrument, or any other source of sound vibrates, and this vibration is communicated to the air, which vibrates with the same rhythm. The air finally communicates its vibration to the drum of our ear. In the

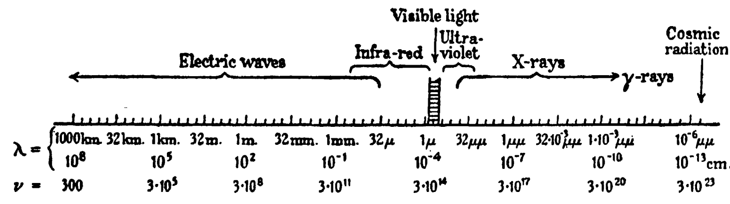


Fig. 20. Wave scale (Lebedew).

case of light, we meet with a difficulty at the outset. The light of the sun and stars comes to us from space. If a sound were made upon a star louder by far than the loudest thunder, we should not be able to perceive it in the slightest, for in outer space there is no material substance capable of carrying the sound waves to the earth. If we are to imagine light as a wave motion in some material substance, this material substance must fill all space. But there is certainly no matter in space, at any rate of density great enough to interfere with the motion of the heavenly bodies. The earth must move around the sun with practically no friction, for since the beginning of history the year has had 365 days. It thus becomes necessary to imagine a new kind of matter which was supposed to fill the whole of space; and this was called the "ether". It had to be so thin and light, that the heavenly bodies could move through it without appreciable resistance.

It had nevertheless to possess the property of being able to vibrate billions of times a second, and to transmit these vibrations at the rate of 300,000 kilometres a second. But this necessitated its possessing elastic properties such as are only possessed by bodies such as steel. In the decades around the middle of last century, endless endeavours were made to find such a substance, or rather to make its existence plausible. But invariably there was something wrong at some point. The ether particles had to be imagined as machines of the most monstrous description, able to meet the most contradictory demands, and nevertheless the explanation always fell short of being perfect.

The conviction grew that the fault did not lie in the feebleness of the scientist's imagination. The English physicist Maxwell, at about the middle of the century, came to the conclusion that it is not possible to explain the properties of light by regarding it as the vibration of a material ether, that is to say, to deduce the science of optics from the science of mechanics. He conceived the brilliant idea of seeking a solution of the problem in quite a different direction, namely in the assumption that optical phenomena are electromagnetic in nature. This was the signal for the complete downfall of the mechanical view of nature. Thanks to Maxwell's work, we find it nowadays much easier to understand electromagnetic phenomena upon their own basis, rather than by attempting to form mechanical analogies to them. Maxwell taught us to look upon optics as a special branch, not of mechanics, but of electromagnetism. Its true nature is revealed to us from the same sources as those from which we understand electricity and magnetism. We will now consider this more in detail.

There is a curious reciprocal relation between electricity and magnetism. We imagine an electric current in a wire as consisting in a flow of electrons. Each electron exercises electric attractive and repulsive forces in its neighbourhood. It carries, as we say, an electric field of force around with it. Furthermore—this great discovery was made by the Dane, Örstedt, in 1820—every moving electron creates around itself a magnetic field. A magnet is thus deflected and moved from

its position when it is in the neighbourhood of a wire carrying an electric current; it is not attracted or repelled, but driven in a circle around the wire in a plane at right angles to it. The magnetic field thus generated is what is called a vortex field. Every moving electron is surrounded by closed lines of magnetic force.

Faraday's great object was to find the counterpart to this. His problem was this: Can one, conversely, produce an electric current by means of a magnet? It is said that he carried a piece of iron and a piece of copper wire for years about with him in his pocket in order to keep the problem constantly before him. The solution which was found by Faraday in 1830 is as follows: If a magnet is made to approach a copper wire in the form of a closed ring, the electrons always present in the wire begin to flow, and an electric current is formed which lasts as long as the magnet continues to approach, that is to say, as long as the magnetic field continues to grow. When the magnet has been brought as close as possible, and the field around the wire has ceased to grow, the current in the wire also ceases to flow. But when the magnet is again withdrawn, a new current flows in the wire in the opposite direction. This can be repeated as often as we like. As the magnet is brought towards the wire and again away from it, a current flows first in one direction, and then in the other. It is in this way that the alternating current supplied by our power stations is produced, the alternating current dynamo at the power station consisting simply of coils of wire close to which magnets are moved by means of steam power.

The important point for us to note is that the movement of the electrons is not brought about by the simple existence of the magnetic field, but by its change. Faraday came to the conclusion that every magnetic field when changing in strength produces an electric field of force in the form of a vortex around the direction of the changing magnetic field.

These two relations between electricity and magnetism were interpreted by Faraday by means of the entirely new notions of electric and magnetic force, of which we have just made use. Physicists at that time did not find it easy to

adopt Faraday's ideas, the more so because this book-binder's apprentice and self-educated man was not able to state his ideas in mathematical terms, although he was one of the greatest experimental geniuses who have ever lived. But he found a disciple in Maxwell, one who possessed the mathematical gift in a very high degree. It was he who clothed Faraday's conceptions of electromagnetic fields of force in mathematical form. This form was given by Maxwell's famous equations. From these the phenomena of electromagnetism can be deduced. They form the mathematical foundation of the whole of technical electricity. Bells and telegraphs, telephones and dynamos, electric motors, and the transmission of power over great distances can all be explained by their means.

The admiration of physicists for these wonderful equations, with their mathematical elegance and symmetry, and their marvellous scientific range has been expressed again and again. The Viennese physicist, Boltzmann, applied to them Faust's words:

Was it a god these characters designed,
The tumult in my bosom stilling
And my poor heart with gladness filling,
With mystic impulse of the mind
The powers of nature all around revealing?

Nevertheless, many of his contemporaries found Maxwell's notions of fields of force very unsympathetic. Their worth had to be tested by the test to which every physical theory must be put, namely, its power to predict the existence of a new and unknown physical phenomenon.

Maxwell made such a prediction, which is of great importance to our present subject. He assumed in his equations that, just as a magnetic field can be set up in space without magnets being present, so it must be possible for an electric field to be formed in space without electrons being present. Then the following should happen. If we cause an electric charge to pass in the form of a spark between two metal balls placed close together, a circular closed magnetic field must arise around an axis through the spark during the passage of the latter; and after its passage, this field must disappear

again. Consequently there must arise in the next moment, around this changing magnetic field, a new electric field, and this again will be the cause of the formation of a fresh magnetic field; thus electric and magnetic fields become linked together like a chain. If the ideas of Faraday and Maxwell concerning electric and magnetic fields are correct, the place where the electric discharge took place should be the source of an electric and magnetic effect or disturbance propagated in straight lines in all directions: in other words, an electromagnetic wave must be produced. If the charge swings several times to-and-fro between the metallic balls, electromagnetic waves having the same frequency will be sent out in all directions. Maxwell was able to calculate what the speed of these waves would be, and he found it to be 300,000 kilometres per second, the same as the speed of light. What if light waves were nothing but electromagnetic waves of this kind, of enormously high frequency? The heated atoms of a red-hot body would then possess electrons rotating rhythmically 400 billion times a second around the nucleus, and this motion would be the cause of the generation of electromagnetic waves, which would be propagated at the speed of light through space in all directions. These waves would then, in falling upon a body, excite the electrons in it to move with the same frequency, and all phenomena of reflection, refraction, dispersion, interference, and diffraction would be produced in the form in which we already know them. Maxwell's equations actually allow us to calculate all these effects.

The majority of physicists were only convinced finally of the truth of the ideas just described when the predicted electromagnetic waves were actually produced. The experimental proof of their existence was given by the German physicist Heinrich Hertz. He succeeded in generating them; and, using apparatus of extraordinary simplicity, was able to prove that they possess all the properties of light waves. They can be reflected, refracted, diffracted, and made to interfere. These waves are known nowadays to everyone by their use in broadcasting. Hertz had no intention of creating wireless telegraphy. It is not thus that inventions are made.

Wireless was developed by inventors by following out the ideas of Hertz. This is naturally a work worthy of all praise: but the whole of technical work, and all those engaged in it would soon be at an end if men did not arise again and again to follow pure science without any practical aim. The aim of Heinrich Hertz was purely scientific. He strove to investigate the innermost nature of the physical world. His experiments prove that the ideas of Faraday and Maxwell concerning the existence of electric and magnetic forces in space, as accurately expressed in Maxwell's equations, fully deserve our confidence. For they were not only able to explain the magnetic and electric phenomena known at the time of their invention, but also to predict an entirely unknown fact, the existence of electromagnetic waves. These were known by their properties to be similar in nature to light waves. The result is to put the phenomena of light into a special department of electromagnetism. Max Planck says: "Maxwell's equations cover all phenomena of electricity and magnetism, and form a bond of union between them: when we adopt them, we feel that we are able to perceive, as from a pinnacle, the unity of nature, at least in this department of science. It is as though we had at least found a fragment of that great formula of the universe, which is the distant goal of all scientific striving."

The only difference between electric waves and light waves lies in the fact that they are of much lower frequency, that is to say much greater wave-length. Those with which Hertz experimented were measured in centimetres and metres, while broadcasting uses waves up to two or three kilometres. The electric waves are thus infra-red waves. It is perhaps more correct to say that all these are electromagnetic waves, but that in one narrow octave, namely that comprised between 3·8 and 7×10^{-5} cm., our eye has been given to us by nature as a sensitive detector. How curious the world would appear to us if we possessed sense organs capable of directly receiving the longer electric waves as well! Since this is not the case, our direct view of the world, unaided by scientific apparatus, is of necessity extremely limited and distorted.

It is very satisfying that we should have thus recognised

the electromagnetic nature of light, the fact that all the manifold varieties of visible and invisible light are similar in nature, since they are all waves which differ only as to length. All these varieties of radiation form a complete and harmonious picture. In future, we shall call them all light, after the narrow little range which we are able to see with our eyes. Only the conceit of humanity, which we find so difficult to get rid of, and which makes us consider ourselves the centre and crown of creation, could lead us to regard that octave of electromagnetic waves which painters use to show us ever fresh aspects of the world, as something very special in nature. At last we have recognised the unity existing between all these different waves from broadcasting to the ultra-violet.

The cathode rays do not of course form part of this range. We have seen that they are not waves. No interference or diffraction effects were known concerning them. Instead, they are minute particles, possessing energy and striking power, and also an electric charge.

How are we to regard those other remarkable rays discovered by Röntgen in the year 1896, and emitted by a cathode ray tube? His discovery was as follows: When the electrons forming a pencil of cathode rays are suddenly slowed down, say by striking the glass wall of the tube or a piece of metal set in their path, a new kind of radiation, the X-rays, is emitted from the point of impact. These rays are invisible, but they act upon a photographic plate, and are able to cause a fluorescent screen to shine. The most striking discovery of Röntgen, a discovery which made them famous from the first, is their remarkable power of penetrating many opaque substances, such as flesh. This property caused them to become quickly known, and in fact indispensable to medicine, as an aid to diagnosis.

The nature of the new rays nevertheless remained doubtful for sixteen years after their discovery, in spite of their widespread use in medicine, and the exact investigations of physicists. Were they corpuscles, or waves? If they were corpuscles like electrons, it should be possible to determine their mass and energy and an electric charge if they possessed one. But experiments corresponding to those made with the

latter failed with the X-rays; for example, they were not deflected by a magnetic field. If they were waves like light, it should be possible to measure their frequency and wave-length, and to diffract them by means of a sufficiently fine grating. But grating experiments either failed or were only very doubtfully successful. By examining their passage through very fine wedge-shaped slits, the conclusion was drawn that if they are indeed waves, they must be at least 10,000 times shorter than light waves. This would make their wave-length of the order of a thousand-millionth of a centimetre. Optical gratings would therefore certainly be too coarse; they would correspond to the garden fence as compared to light. No diffraction experiments with them could possibly succeed. The construction of gratings with 1000 or more parallel lines to the centimetre ruled by a diamond upon a glass plate is already a masterpiece of technical skill; the construction of such gratings with 10,000,000 lines per centimetre would be impossible. Where could such gratings be found? The distance apart of the rulings would have to be, as we have seen, but little greater than the supposed wave-length, that is to say of the order of a hundred-millionth of a centimetre.

In the year 1912, the German physicist von Laue conceived a brilliant idea which was carried out experimentally by two of his students, Friedrich and Knipping. Von Laue proposed to use crystals as diffraction gratings. In the first place, crystals, if our notions of the atomic structure of matter are correct, must have their atoms arranged with perfect regularity according to a geometrical principle, as we have already seen in Fig. 4. At the time this idea was rather daring, since the atomic structure of matter was by no means a settled fact, whereas von Laue assumed it as self-evident. We may recollect the diffraction images given by a grating consisting of a system of parallel slits, and also those given by two gratings with their lines crossed, that is to say, a collection of slits arranged in a certain way upon a surface. In the case of a crystal, we should be dealing with many such surfaces, set one behind the other, in other words, a grating in three dimensions. But something else is also essential. The spacing

of the grating must be of the right size, otherwise the crysta will be no more use to us than a garden fence is in dealin with light. We know that its spacing must be about on hundred-millionth of a centimetre. This, as we know, is exactly the size of atoms, and it is not likely that the distance apart of the atoms in a crystal will be much larger than this, since they must be close neighbours. We are able to calculate mathematically the diffraction images which should be given by such a three-dimensional crystal grating, just as we are able to calculate them for the ordinary optical grating. The bright spots resulting from the interference of rays in the same phase lie upon the points of intersection of ellipses with hyperbolas or parabolas. Figs. 21 and 22 show us the expected result, and that actually found. The lattice structure of the crystal is revealed in its wonderful symmetry, and the nature of this symmetry can be determined. Photographs of this kind have become one of the most important supports of the atomic theory. The mineralogist uses X-rays to decipher the way in which the crystals of minerals are built up from their constituent atoms, and X-rays have conquered a fresh field in the testing of technical material. The metallurgical industry, and the textile manufacturers also, are beginning to realise the possibilities which lie in making Laue diagrams of their materials by means of X-rays; they can draw conclusions concerning the fine structure of their materials, and hence concerning such properties as strength and extensibility.

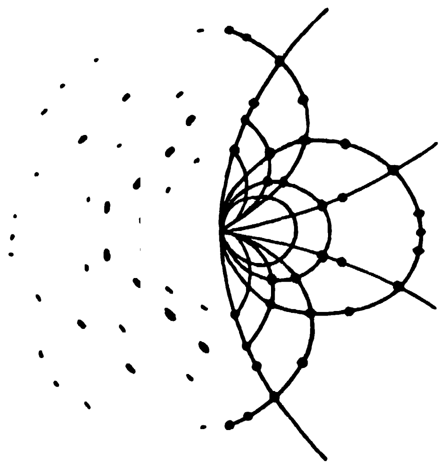


Fig. 21. Diffraction pattern produced by X-rays falling on a crystal. On the left is the experimental result, on the right the theoretical construction according to von Laue.

Furthermore, these photographs give us the proof tha

Fig. 22. Diffraction of X-rays by a crystal; photograph by von Laue

[*to face page 50*.

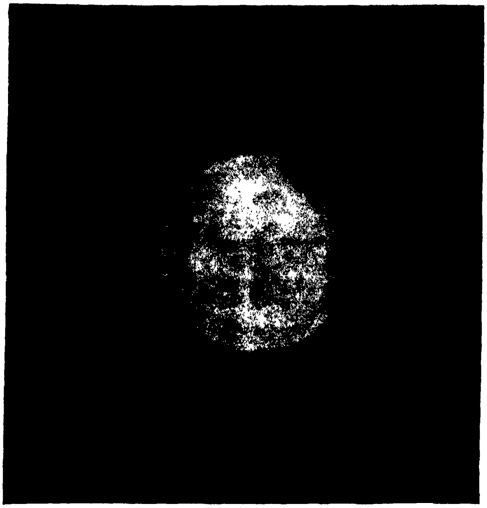


Fig. 23. Diffraction image produced by X-rays passing through crystalline powder.

[*to face page 51*]

X-rays are also electromagnetic waves. Their wave-length can be calculated from the photograph to be about 10,000 times smaller than that of light. If we take, in place of a single thin crystal plate, a quantity of finely powdered material in which the single crystals are arranged entirely irregularly, we get the picture shown in Fig. 23.

Photographs of this kind can also be used to calculate the wave-length and frequency of X-rays.

Finally, the gamma rays emitted by radium are also X-rays, of extremely short wave-length.

We can now summarise what we have been saying. The physical world as seen by classical physics is occupied by two quite different kinds of entities. On the one hand we have matter, which can be divided up into ultimate units, protons and electrons, each of which is characterised by a certain mass m, impulse p and energy E. As opposed to matter, we have light, which is emitted and absorbed by matter. It consists, as diffraction experiments clearly show, not of minute particles, as does matter, but of electromagnetic waves; the empty space of the universe, and also the space between bodies is capable of conducting varying electric and magnetic forces with a velocity of 300,000 kilometres per second. Such a wave is characterised, and its properties known, when we know its wave-length λ and its frequency ν.

This picture formed by the physicists of the material world has without a doubt one great drawback as compared with the picture formed by the non-scientific man in the street. It is much more abstract. A table, a piece of paper, no longer possess that solid reality which they appear to possess; they are both of them porous, and consist of very small electrically charged particles, which are arranged in a peculiar way, as we saw in the first chapter. The beautiful and varied colours which rejoice our hearts in daily life appear to the physicist to be merely the result of our subjective nature. All that exists outside ourselves is a system of electromagnetic vibrating fields each possessing a characteristic frequency and wave-length. But, on the other hand, this world of the physicist is nevertheless richer than that of the man in the street. The physicist sees connections never dreamt of by

the layman, for example the community in nature of radio waves and light waves. This perception is mathematical in character. The physicist's picture also effects much more. It alone is what suggested to Rutherford the transformation of nitrogen into hydrogen, and it teaches the technical expert how to construct apparatus enabling us to sit at home and listen to what is being said or sung in other parts of the earth. The common man's view of the world endows him with no such powers.

One might therefore be tempted to sing the praises of physical science very loudly indeed, and rejoice concerning our almost miraculous advance. We are preserved from this kind of conceit as soon as we pass from a consideration of matters which lie within the classical scheme of things, and examine that new territory into which the science is advancing to-day. The present-day physicist has the good fortune to take part in this rapid and exciting drive.

While the picture formed by man of the world around him does from time to time acquire the appearance of being rounded off and complete, history tells us that it never includes more than a small part of the wonders of the real world, and that we soon are driven to abandon such harmonious pictures, on account of disharmonies which intrude themselves upon us, and lead to new investigation. We will now pursue systematically the facts which have undermined the classical picture of the world. Since it was constructed, new discoveries have been made, and we are called upon to find an explanation for these. A new and more profound picture of the world can only be our reward if we follow, without any preconceived ideas, the exciting path taken by physics in the last generation.

CHAPTER III

Light Quanta

In the central region of the natural sciences, where the concept of "action" is to-day displacing the concept of energy, where the concepts of space, time, and mass are leading to a new revelation concerning that mystery, our latest name for which is "matter", in this region where only that exists for us that can be measured, precisely there do we find arising out of the fog of theories, like the light of the ancient and eternal day, Plato's vision of a numerical theory of nature, and with it the wisdom of Pythagoras.

—HUGO VON HOFFMANNSTHAL.

(Speech made at a meeting in favour of classical education.)

Max Planck says: "In the meantime problems have arisen which probe more deeply into our whole mode of physical thought than we should perhaps ever have thought possible."

At the outset we dealt with matter, and saw that it was made up of a mosaic of tiny particles, the protons and electrons. Our last chapter taught us that besides matter we find another entity, light. This comprises all electromagnetic radiations. A large number of very varied phenomena are thus included, but they are all alike in nature, and consist in electromagnetic waves. In order of decreasing wave-length we are familiar with a whole range of electric waves, such as are received in broadcasting; next, shorter in length, we have the infra-red heat waves; then visible light ranging through all the colours of the rainbow; then ultra-violet light, familiar to us from "sun-ray" lamps; and finally X-rays, the shortest of which are identical with the gamma rays of radium.

While the wave theory of light celebrated a further triumph in the year 1912 by explaining the nature of X-rays, a discovery had been made twelve years previously which was destined to endanger it in the most unexpected manner, and on the very ground where it ought to have been most secure.

the case of light of each wave-length, but that they could be calculated from the formula:

$$\epsilon = h\nu,$$

where ϵ (epsilon) is the quantum, ν is the frequency of the light and h a universal constant of nature, known as Planck's radiation constant, and having the value $6{\cdot}55 \times 10^{-27}$. The *earth has a weight about equal to 10^{27} pinheads. Planck's constant has about the same ratio to the number one that six pinheads have to the weight of the earth.* The energy quanta vary according to the frequency; the frequencies of visible light, as we see by referring to the table, Fig. 20, vary between 4 and 7×10^{14}; thus the corresponding quanta vary between 25 and 46×10^{-13} units of energy, known as "ergs". Here one erg represents the amount of energy necessary to lift a pinhead up a distance of one centimetre. The energy quanta of visible light are thus only a few billionths of this very small unit.

These energy quanta are therefore extremely small. For visible light they are larger than for heat waves. Those of ultra-violet are still larger, and those of X-rays ten thousand times larger still, that is to say, of the order 10^{-9} ergs and more. The steps between the different amounts of energy which it is possible for an atom to take up are therefore large in the case of light of high frequency, become smaller in the case of red light, and smaller still in the infra-red, where they become immeasurably small as regards all our experiments. Hence, in this region, the emission and absorption of light can be assumed with sufficient accuracy to be continuous, and we are able to apply the classical theory, and do without the quantum. But as soon as we deal with light of a greater frequency and shorter wave-length, its absorption and emission can only be explained by exchange of energy taking place discontinuously, that is to say, with the aid of the entirely new constant of nature h. The science of physics has more and more come to regard an understanding of the true inwardness of this "quantum of action" as one of its principal problems. But at this point in our examination, its meaning appears to be that of a measure of the steps in which energy is exchanged

with the atom. It is, so to speak, the smallest coin with which the atom can pay out or receive its energy; the value of this coin depends upon the wave-length of the radiation which transmits the energy. As we proceed, we shall become more and more conscious that action itself has an independent meaning alongside energy. It was only because these coins with which energy is paid out and received are so small that they had hitherto not been noticed, just as one might take a staircase with very minute steps to be simply an inclined plane.

The quantum hypothesis thus leads to correct results. But are we justified in regarding it as something more than a mere source of correct formulae? Does it actually lead us to a deeper insight into the workings of nature? Planck said in an address which he gave when he received the Nobel prize: "Either the quantum of action is only a fictitious magnitude, and the whole deduction of the law of radiation more or less of an illusion, little more than a game played with formulae, or the deduction of the law is based upon a true physical idea. In the latter case, the quantum of action is destined to play a fundamental part in physics. In it we have found something quite new and unprecedented, which is certain to entirely revolutionise our physical thought; for since the foundation of the differential calculus by Leibniz and Newton, this thought has been based on the continuity of all causal connections in nature."

Planck had been cautious, and had stressed only the fact that, in order to agree with experimental results, it is necessary to assume that the emission and the absorption of light takes place discontinuously in quanta. But he made no statement concerning light while travelling from one place to another.

Let us consider this point a little more carefully. According to classical physics, light, as we have seen in the last chapter, arises from the fact that the electrons in the atom oscillate or revolve rhythmically 400–800 billion times a second, and in doing so, send out electromagnetic waves of the same frequency; these waves we call light. As long as the electron is in motion (and it must be moving continuously, otherwise it would be attracted and swallowed up by the nucleus), it must

continuously emit light without the smallest interruption, *this light having a frequency determined by the rhythm of* the electrons' motion. Now this does not agree with the quantum hypothesis, according to which light is emitted discontinuously in quanta. Matters become still more difficult when we try to imagine how absorption takes place. Let us suppose that a lamp has emitted a quantum of light; the energy thus emitted must then travel out from the lamp in the form of waves in all directions, and thus be spread over spheres of ever increasing diameters. How then can a whole quantum arrive at a particular point of the retina of the human eye, where of course, the absorption of light also takes place only in whole quanta? The question is very hard to answer. Are we to suppose that the first wave waits as it were, at a closed door until sufficient trains of waves have followed it up to accumulate a whole quantum at that point, whereupon the door opens and the light is absorbed? This is contrary to experience. When the frequency, and therefore the quantum is large, as in the case of X-rays, it is easy to calculate that a considerable time would be required for a quantum to collect. But all our experience teaches us that the absorption of X-rays takes place instantaneously; and what are we to say when the supply of X-rays is stopped before a quantum can have accumulated? No other conclusion is possible than that the wave theory, which agrees so well with the phenomena of diffraction, cannot be made to agree with the experimental evidence which we have of emission and absorption by quanta. Planck's discovery that matter only emits and absorbs light in quanta is not therefore a simple new fact like many another, but rather a dangerous explosive which is capable of destroying the whole structure of the wave theory, which once seemed absolutely secure.

Hertz had said ten years previously: "One cannot study Maxwell's wonderful electromagnetic theory of light, without occasionally feeling that the mathematical formulae possess life and reason of their own, as if they were cleverer than we, cleverer even than their inventor. They seem to give us more than was actually put into them at the time of their invention." This beautiful harmony of our views of electrical

and optical phenomena has been destroyed; we have been forced to realise that it was a deceptive picture, and to accept in its place a strident and inexplicable discord.

The first to take a bold step forward was Einstein in the year 1907. He proposed to attempt to give up the wave theory of light entirely, and to assume that the light quanta, when radiated, remain permanently together without spreading. According to this "light-quantum hypothesis", light is not propagated in waves, but in quanta of the value $E=h\nu$, as "light-darts" or "photons". This makes it easy to understand how a quantum radiated from a lamp is instantly absorbed when it strikes the retina. This conception is quite familiar to us, for it is nothing more than Newton's corpuscular theory, only that the corpuscles are now no longer material in nature, but simply little lumps of energy. But this new energy corpuscle meets with exactly the same insuperable difficulty as Newton's theory when it comes to explaining how light can be bent to one side in passing through a narrow slit. For this reason the theory was again abandoned. It is also impossible to conceive how these light quanta could travel as waves, for even in that case they would go straight through the spaces of a grating just as if they were corpuscles. No explanation is given of how light added to light can in certain circumstances result in darkness, and how light can be turned sharply aside from its path.

The quantum theory is not thus a simple question in one department of physics; it raises a fundamental problem. We have before us two theories of light, each of which is able to explain only a part of what we know about the properties of light. Boltzmann, whom we have already quoted, once said concerning another problem of physics: "These are truly interesting questions, and one must always regret that one will not live to see them decided. O overweening mortal! Your fate must be to experience the joy of the struggle." These words are true of ourselves in the present case.

We may now ask whether the light quantum hypothesis is only able to explain one set of phenomena, those concerned with radiant heat. In that case, we should not perhaps lay too much stress upon it. Can we find other phenomena which

afford a proof that light consists not of waves but of corpuscles? Let us recollect our fluorescent screen. It either threw back light of the same colour, or of lower frequency. Even X-rays, in spite of their very high frequency, can be made visible to our eyes by the fluorescent screen, which transforms them into visible light, that is to say, waves of much smaller frequency. We always find that Stokes's law holds good; monochromatic light (that is, light of a single frequency) is always reflected by matter without change unless the material is fluorescent, when the colour is changed to a lower frequency. The wave theory is not able to explain this rule of Stokes. What has the quantum hypothesis to say about it? It says that light is received by a material surface in the form of quanta. Either these quanta are thrown back like billiard balls from a cushion, unchanged, and then produce, in our eyes, the same effect as before they struck the surface, or a part of the incident light energy is absorbed; the reflected quantum $h\nu$ is then smaller. Since the constant h has not been changed, the frequency ν must have become smaller. This is exactly what Stokes's law requires.

It is a remarkable fact that light of higher frequency, say blue light, carries with it more energy than red light of a lower frequency. We know that the action of light on the photographic plate is produced by that of high frequency. Any amount of red light will not fog our plates in the dark room, but the fraction of a second's exposure to violet light is enough to alter immediately the bromide of silver film. The surprising fact appears that this same result is obtained in the photoelectric effect, which was discovered by Hertz, and investigated by Hallwachs and Lenard. The effect is quite incomprehensible on the classical theory, but easily explained by light quanta. If we take a gold-leaf electroscope, a well-known apparatus for indicating electric charge (Fig. 24), and touch it with a rod of ebonite which has been rubbed with a cloth, an excess of electrons flows into it, and it becomes negatively charged. The gold leaves diverge. If we now expose the electroscope to the light of a lamp, we see the leaves fall together. The electrons escape. The energy of the light falling on the electroscope is transformed into energy of

motion of the electrons. Up to this point, there is nothing very strange about the phenomenon. But according to the quantum conception, the light must offer its energy to the electrons in whole quanta of the value $h\nu$, so that something very curious may be expected. The higher the frequency of the light, the greater is the energy of the quanta, and the electrons should therefore be given a higher velocity. This is actually found to be the case, and the classical theory is unable to give us any explanation of it. It would of course teach us to expect the action to become stronger when the source of light is made brighter, or brought nearer. We should, of course, expect the same effect on the basis of the quantum theory, for more photons would reach the electroscope. But

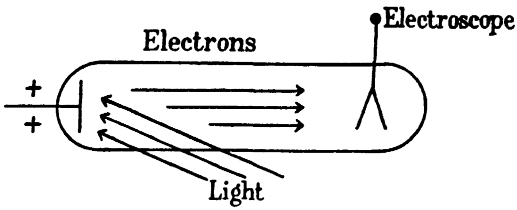


Fig. 24. Release of electrons by monochromatic light (Millikan). The electroscope is an insulated metal rod, from which hang two gold leaves. When the rod is charged, the gold leaves repel one another, and diverge.

the classical theory gives us no reason to expect blue light to be more effective than red; as, for instance, a distant bluish-coloured star to produce more effect with its extremely feeble light than a strong yellow lamp close at hand. Nevertheless, experiment shows this to be the case.

These experiments were greatly improved in the year 1916 by the American physicist, Millikan. He illuminated a disc of metal with monochromatic light of a definite frequency, and measured the energy of the electrons emitted by causing them to fall upon an electroscope. When he then applied to the disc a weakly positive electric voltage (Fig. 24), the slower electrons emitted were held back, and the electroscope no longer showed so strong an effect. He then raised the voltage applied to the plate until even the fastest electrons failed to arrive at the electroscope. In this way it was possible

to measure the energy of the electrons, and Millikan was able to prove that the energy of the fastest of these increased exactly in proportion as the frequency of the light is increased. The energy of a single photon is changed into energy of motion of a single electron. Millikan was thus able to prove the existence of single light quanta. By means of the frequency ν of the light used, and the energy of motion of the ejected electrons, he was able to calculate the constant h. He found precisely the same value as that calculated by Planck from quite different measurements. Millikan himself was astonished, and closed his paper by saying that he still regarded the physical theory on which the calculation was based as quite untenable. He meant that the light quantum theory could not be reconciled with the wave theory. However that may be, the results are the best possible proof of the fundamental assumption of the quantum theory, namely that the energy of vibrating electrons is emitted discontinuously. They form the most direct and striking proof we have hitherto obtained of the true physical reality of Planck's constant. A few years later Einstein was given the Nobel prize for his "quite untenable" theory. And the well-known physicist Sommerfeld said: "No doubt exists that we are here dealing with one of the most fundamental of natural laws." The existence of quanta is thus an assumption based upon measurements, and as Planck said: "What can be measured must exist."

The converse experiment also succeeds, as was shown by Franck and G. Hertz in 1913. They allowed electrons to escape from an electrically heated wire: these charged a metallic disc placed near the wire positively, the electrons striking it with a measurable energy. The intermediate space was filled with mercury vapour. They then applied an increasing voltage to the wire, and thus increased the energy of the electrons. At first nothing remarkable happened. The electrons shot unhindered through the mercury vapour to the disc with increasing speed. But quite suddenly, when the energy of the electrons reached a definite value, only quite slow electrons arrived at the plate, but the mercury atoms began to glow. The atoms were thus able to absorb this particular amount of energy, and therefore robbed the

electrons of it. It was not sufficient, as in the former case, to tear electrons out of their positions in the atoms, but the latter were thrown into a state in which they possessed more energy. From this state, they were able to fall back into their original state of less energy. In doing so, each atom emitted the energy which it had just absorbed in the form of a photon $h\nu$; the mercury atoms, in other words, emitted light of this frequency ν.

The experiment reminds us of an automatic machine. If this is built to take shillings, a sixpence or a farthing will not cause it to deliver a packet of cigarettes; the coins simply fall through the machine. But it takes our shilling and gives us a packet in exchange. This packet is the light radiated.

If we now measure the frequency of the photons, and the amount of energy which they take up from the electrons, we can again calculate a value for Planck's constant h. We get the same result as in the other two cases. If we put still more energy into the electrons, the colour of the light emitted by the mercury atoms does not at first change, but as the energy is increased further, they suddenly begin to emit light of a higher frequency ν', that is a new and larger quantum $h\nu'$, corresponding to a greater energy which they are now able to take from the electrons. It is as though we were dealing with a modern slot machine which is able to take various coins in the same slot and supply us with packets of various sizes according to the value of the coin. This fact that atoms are not able to absorb energy in any amount, but only in definite quanta, and then radiate light of definite colour, is quite incomprehensible according to classical ideas. Franck and Hertz also received the Nobel prize for their measurements. Their method also makes it possible to determine the energy differences of the various, uniquely possible states of the atom.

If faster and faster electrons are used, we get, as each new quantum step is reached, light of ever increasing frequency, that is to say, farther and farther in the ultra-violet. When the voltage is no longer a few volts, as in the case of the experiments of Franck and Hertz, but tens of thousands of volts, we should expect light of ten thousand times the frequency, that

is to say, X-rays. And in actual fact, voltages of this magnitude are needed to produce X-rays in the ordinary X-ray tube. When we have comparatively low voltages, we get first of all the "soft" X-rays, and as we increase the voltage, we find, as is to be expected, that the rays get harder and harder, that is to say, shorter and shorter in length.

This dependence of the hardness of an X-ray tube upon the voltage applied to it can only be explained by the quantum theory. A definite voltage can only produce X-rays of a corresponding definite maximum frequency, and this frequency, and voltage, can again be used to calculate accurately the Planck constant; the result is again in excellent agreement with previous results. The fastest electrons of all, the beta rays, also generate the hardest X-rays, namely, the gamma rays.

Throughout the whole range of the spectrum, starting with the infra-red heat rays with which Planck concerned himself, and continuing through to the gamma rays, the exchange of energy in quanta is a fact as certain as any other in physics, and it is irreconcilable with the wave theory of light.

But the extraordinary range of the corpuscular hypothesis is most clearly seen in the case of the famous effect discovered by the physicist Compton in 1923. The light quantum hypothesis requires us to regard light as composed of minute particles, which move with the velocity of 300,000 kilometres per second. If they are particles, they must, like all particles, possess mass, energy and impulse, as we have seen in the first chapter. Their energy must be given by Planck's fundamental equation $E=h\nu$. According to the theory of relativity, the energy of a body is connected with its mass by the equation $E=mc^2$, and hence $h\nu=mc^2$. This allows us to calculate the mass of a photon as

$$m=\frac{h\nu}{c^2}.$$

This fraction is extremely small; for the very small number h, though it is multiplied by the large number ν, is divided by the still larger number c^2. But how can photons possess mass if they are light and not matter? But not only ordinary

matter possesses mass, that is to say inertia, resistance to forces tending to accelerate or decelerate; the material of which light is composed must also possess inertia, and hence also impulse, power to deliver a blow. It must be possible to devise an experiment to show this hitting power of light quanta. We can calculate the impulse; it is $p = mv$. Since

$$m = \frac{h\nu}{c^2}$$

and the velocity of the light corpuscles is c, we have

$$p = \frac{h\nu}{c^2} \times c = \frac{h\nu}{c}.$$

Since $$c = \lambda\nu, \quad p = \frac{h\nu}{\lambda\nu} = \frac{h}{\lambda}.$$

The striking power of the photon is thus always very small on account of the smallness of h; it is relatively greatest when λ is smallest, that is to say, in the case of light of short wave-length, such as X-rays.

The impulse must make itself felt when the photons strike obstacles, that is when light strikes upon matter. What does the classical theory lead us to expect? Matter consists of atoms which contain vibrating electrons. Each electron has its own natural period of vibration. But light is an electro-magnetic wave of a definite frequency. It sets into vibration those electrons which themselves are tuned to this frequency, and these again naturally emit light of the same frequency. The two sets of waves overlap. Their interference results, as can be shown, in a single wave, moving in the direction given by the law that the angle of incidence is equal to the angle of reflection. There is no change in the colour, that is to say, the frequency, of the light.

But according to the quantum theory, the photons must be pictured as particles in motion. Now the photons of visible light are bodies with very small impulse. As compared with them, the particles of which the atom is composed, the protons and electrons, are quite heavy bodies, and hence will no more move as the result of being struck by a photon, than will the earth in colliding with a football. The photons will

picture of the world had been carried in classical physics, and had been developed beyond the notions suggested by the direct impressions of our senses. The world is not a human being, and not the creation of such a being. We may be compared to an ant-heap or bees in a beehive, attempting to form a picture of the world. Such an attempt is obviously doomed to failure from the start.

But mankind will not give up the struggle; each generation forms a new picture of the world. Every one of these pictures, even the latest, is no doubt far removed from reality, but each is better than its predecessor, and the time that has been spent on constructing them appears to us to be well spent, as well spent as the efforts of the poet and the musician, who capture the world by means of their artistic symbols. Our attempt cannot be a complete failure, for the reason and imagination which we use in making it, are themselves a part of the world, since they are part of ourselves. We do manage to capture a part of the truth. The truth which man can grasp consists in shadow pictures of reality. It is thus that Plato compares, in a famous passage at the beginning of the seventh book of his *Republic*, man in search of knowledge to a cave dweller imprisoned in his cave, able to perceive what is happening outside the cave only by means of shadows falling upon its walls through a small opening.

In the most favourable case, the physical picture of the world at a given period presents a summary of experimental knowledge free from internal contradiction; and the classical picture existing shortly before Planck's was almost of this character. But no such picture can take account of facts of nature not yet discovered by man. The search of the truth is carried on in small steps, and is without end; the theoretical ideas of the physicist die and are born whenever important new experimental discoveries are made. The discoveries of the experimental physicists are immortal, discoveries such as the diffraction of light by narrow openings, or the step-by-step emission of light of different colours which takes place when mercury atoms are bombarded with electrons of different speeds, as we have just described in this chapter. All these are experimental facts which cannot be lost to us, and

always remain true. It is only our interpretation of these facts which passes and changes. Would it not then be better for physicists to abandon all attempts to interpret such facts? No; for in the first place these attempted interpretations are, after all, the physicist's best gift to the world, the quintessence of his teaching. On the other hand, this picture of the world which his imagination creates is the strongest spur to new discoveries. The goal of his work is the making of a new and better picture, one more close to the truth, one fitted to spur him forward once more. The road to the truth sought by civilised white man is endless, and he can never traverse more than a mere fraction of it.

At the present time we are confronted with two contradictory theories, the wave theory and the quantum theory of light; furthermore, we have been quite unable to find any explanation of the atomistic structure of action. This is a proof that some essential falsity must underlie our physical concepts. The error which must be present in the fundamental assumptions which have hitherto guided physical discovery, must be unearthed, and the foundations of our science must be rebuilt. This object has been energetically pursued by the most famous physicists of all countries for the past twenty years. A whole generation of workers has devoted itself to this problem, the problem *par excellence* of physics, with what might almost be termed a one-sided passion. For the solution of this problem promises to raise our physical knowledge to an entirely new level.

This becomes clear to us when we consider the new physical magnitude, Planck's constant of action, which has been the cause of the whole revolution. This physical magnitude action, which Planck taught us could only exist in multiples of a certain small minimum amount, appears to be a quantity of a higher order of reality than most other magnitudes which occur in physics. This can be seen from the following consideration. The energy $E=h\nu$, that is to say, $h=E\frac{1}{\nu}$; the frequency ν means the number of oscillations taking place in a second. Hence $1/\nu$, the time taken for a single complete vibration, is measured in seconds; the measure of the

quantum of action h is therefore energy $\times$ time. Energy is a quantity of the nature of space; hence, in the quantum of action, we are dealing with something which is measured by space $\times$ time. The theory of relativity teaches us, as we have already said, that we can only arrive at views of full general validity when we cease to regard events as divided into time and space, but consider instead a four-dimensional world combining together space and time. The concept of action, which belongs in this four-dimensional world, thus possesses the validity which attaches to this world. It is not something which we can readily comprehend, as we can all that we are able to analyse into space and time, but belongs to a deeper and more comprehensive region of truth. Planck's recognition of the fact that it must be regarded as atomistic in nature is a discovery which probes the deepest regions of our previous physical knowledge. This discovery will bring about more important changes than we have hitherto considered in this chapter. Until it was made, no success was obtained in reconciling the contradiction between optics and quantum physics. Science had not reached the point at which this could be done. Before we proceed to consider this latest development, we will first become acquainted, in the next two chapters, with the magnificent success obtained by the light quantum hypothesis in its application to the structure of atoms.

We still need to discuss a particular kind of radiation, which is known as the cosmic or ultra-radiation, discovered by Hess in the year 1912. It is found everywhere on the earth, and can be detected by its power of ionising gases. It is able, like X- and gamma-rays, to split up gas molecules into electrically charged particles, or ions. The power which this radiation possesses of penetrating layers of water and air of various thicknesses has been measured. Balloon journeys were made into the stratosphere, a height of 12 miles having now been reached by passengers; also, balloons loaded with self-recording apparatus for measuring ionisation but without passengers, are sent up. On the other hand, similar apparatus has been let down into very deep lakes. In all these cases, a sensitive self-recording electrical apparatus measures the

number of ions formed per second by the action of the cosmic radiation upon the gas enclosed in the apparatus. The results prove that the penetrating power of cosmic radiation is quite extraordinary. Physicists are not yet sure whether it consists of light of extremely short wave-length, or of high-speed material particles, or of a combination of the two. Since cosmic radiation is found to be stronger the greater the height above the earth's surface, it seems probable that it does not originate in the earth, but comes to us from outer, "cosmic" space. Regener's measurements have recently shown that we can distinguish in it five kinds of rays differing in penetrating power. If we assume that these rays are not material in nature, but are of the nature of light, we can, granted certain assumptions, calculate their wave-length from their penetrating power. The two hardest rays would then be of wave-lengths $13{\cdot}7 \times 10^{-14}$ cm., and $3{\cdot}28 \times 10^{-14}$ cm. This radiation would thus have a wave-length only one-thousandth part of that of the hardest known X-rays (see Fig. 20, p. 42).

The search for a possible source in outer space for this extremely hard radiation has brought forward the suggestion that it may perhaps be produced by the transformation of matter into radiation; the possibility of such a transformation has long been suspected by astronomers. What matter would be concerned in this transformation? The energy of the photons of the cosmic rays would have to be equal to the energy mc^2 of the matter from which they were formed. Now $mc^2 = h\nu$, so

$$m = \frac{h}{c^2}\nu = \frac{h}{c^2}\frac{c}{\lambda} = \frac{h}{c\lambda}.$$

Hence we get for the value of m in the case of the cosmic ray of wave-length $13{\cdot}7 \times 10^{-14}$,

$$\frac{6{\cdot}55 \times 10^{-27}}{3 \times 10^{10} \times 13{\cdot}7 \times 10^{-14}} = 1{\cdot}6 \times 10^{-24};$$

and similarly for the other ray of wave-length $3{\cdot}28 \times 10^{-14}$,

$$6{\cdot}6 \times 10^{-24}.$$

We may recollect (p. 14) that the first number is the mass of an atom of hydrogen, and the second roughly four times

as great, and thus equal to the mass of the atom of the second lightest element helium. This double coincidence, leads us to imagine that the two most penetrating components of the cosmic rays may be extremely hard X-rays which reach us from distant worlds in space, where hydrogen and helium atoms are being turned into radiation.

Other components of cosmic radiation appear to originate in an opposite type of process, the formation of more complicated atoms from simpler ones, as Millikan and Cameron in particular have shown. When four protons join up to form a helium nucleus, the mass of the product is smaller than the mass of the four hydrogen nuclei out of which it is made, as we can see by looking at the table of atomic weights (Fig. 3, pp. 16, 17). But mass is a form of energy. Hence if the mass decreases by an amount m, energy amounting to mc^2 must appear elsewhere, since the total energy remains the same. If we assume that the lost mass appears as radiation, we are at once able to calculate the wave-length. We do in fact find it to be exactly equal to another of the experimentally discovered components of cosmic radiation.

[*Translator's note:*

This matter of the cosmic rays is one of the most important and exciting in physics at the present time. Since the German edition of this book, what is said above has already been superseded by views based upon further experiments; nevertheless, it is as well that the reader should be acquainted with speculations which a year or two ago were widely discussed. The change of view has arisen from a fuller knowledge of the distribution of the rays over the earth's surface, gained by experiments carried out by a large number of observers both on the surface, and at high altitudes. It is now established that the rays come to us from all directions of space in practically equal intensity; they cannot therefore originate in our system of stars, the Milky Way, but can only come from the distant nebulae, if indeed they are being produced at the present time. On the other hand, their intensity is less at the equator than at the poles, and this difference is greater, the higher above sea level the measurements are made. This fact can only be explained as due to the action of the earth's

magnetic field upon the rays, which must therefore consist, like the alpha and beta rays, of charged particles of varying energies. There are now believed to be three, or perhaps four, kinds of rays.

Calculation shows this explanation to be fully consistent with their consisting of alpha particles, electrons, and protons, moving at speeds giving them energies thousands of times greater than those of the alpha and beta rays of radioactive bodies. These speeds must be less than a foot per second smaller than the velocity of light, the speed at which the mass, and therefore the energy, of a moving body, no matter how light, becomes infinite. The transformation of atomic nuclei now seems to be inadequate to account for these enormous energies (up to 10^{12} electron volts). When the rays strike atomic nuclei, "showers" of 20 or more high speed positrons and electrons are produced. This is believed to be due to the production of very high-energy photons by the collision, which photons are then converted into "electron twins" (see p. 201).

Various new theories of the rays are being discussed. One of these is connected with the fact that the Universe appears to be expanding very rapidly; the most distant nebulae appear to be rushing away from us at incredible speeds. This argues an initial explosion of some sort as the send-off for our world as we know it, and Lemaître suggests that the cosmic rays were produced in this explosion, and have been in existence ever since.

The total energy received by the earth from these rays is only somewhat less than that received from starlight. The ionisation they are continually producing in the air is about half as much as that due to diffused radioactive material—three ions per cubic centimetre per second. So they are a new factor which will need to be taken into account in all future speculations concerning the past and future of the dead—and living—world. Who can say what the effect of a cosmic ray hitting a gene may be, at the moment when the whole future of a living creature is being decided by that gene?]

CHAPTER IV

The Structure of the Hydrogen Atom

...on looking back with some sense of perspective, we cannot fail to recognise that the last Liverpool meeting marked the beginning of what has been aptly termed the heroic age of Physical Science. Never before in the history of physics has there been witnessed such a period of intense activity, where discoveries of fundamental importance have followed one another with such bewildering rapidity.

—Sir ERNEST (now Lord) RUTHERFORD, Presidential Address to British Association, Liverpool, 1923.

Let us recollect the peculiarly individual manner in which atoms react to light of different colours when it falls upon them. They allow the great majority of all colours to pass by them with complete indifference, but absorb a few colours having a quite definite frequency, and therefore a quite definite energy $h\nu$; and these colours are precisely those which they are able to radiate. This leads us to conclude that an atom when in a stable condition cannot take up any arbitrary quantity of energy, but only quite definite amounts. An atom is thus not able to exist in an unlimited number of energy states with their energy continuously graded; it changes its energy state quite suddenly, switching between a certain limited number of possible states. It is like the automatic machine which is only arranged to take coins of certain denominations. Such a machine can only contain sums of money which can be made up out of these coins, since it rejects all other coins which are put into it. An atom changes from one possible energy state to another by absorbing or emitting a light quantum of the correct amount. In classical terms it is only able to absorb or emit light of quite definite colours, that is to say frequencies. By means of the spectroscope, the colours of light peculiar to each atom can be determined by measuring the emission and absorption spectra. These must afford us a

means of investigating the structure of atoms, and of obtaining a better understanding of quantum phenomena.

The laws according to which light is absorbed and emitted by atoms are very accurately known. The process takes place in a pure form only when the atoms are in the gaseous state. In the liquid and solid states, the atoms are close enough together to affect one another in a very complicated fashion, and the various possibilities of energy changes are very numerous. The number of lines which appear in the spectrum becomes so great that no gaps between them can be detected, and we have what is called a continuous spectrum. We can easily examine such a spectrum by looking at the light of a burning candle, in the flame of which are white-hot particles of solid carbon, or the white-hot filament of an electric lamp, through a spectroscope. We see a complete spectrum of all the colours of the rainbow, without any gaps at all, and if we use suitable apparatus, we can also detect the ultra-violet and infra-red rays. But when we have a glowing gas, we are able to distinguish the optical behaviour of the single atoms. The gas can be made to glow by heating it sufficiently. If we hold a copper wire in a flame, the copper evaporates, and its atoms glow and send out the green light which characterises them. If we throw a trace of common salt, soda, or Glauber's salt, all of which are compounds of the metal sodium, into a gas flame, it is coloured a brilliant yellow. In every case this yellow sodium light has a frequency $\nu = 509 \times 10^{12}$ per second, as is proved by observation of it through a prism. The yellow light is emitted by heated atoms of sodium, quite independently of the compound in which they are contained. Similar effects are given by all atoms. The brilliant yellow-green light emitted by the atoms of barium, and the splendid red light of strontium, are well known to us from their use in fireworks.

These colours are so characteristic for each element, and this method of investigation is so sensitive, that it is used by chemists for recognising minute traces of an element in any of its compounds. The eye is not of course able to determine whether the colours are pure, or mixtures of several colours. The light emitted by the glowing atoms must be split up by

means of a prism or grating. It then can be seen that the green of copper or barium and the red of strontium consist of single lines, the frequencies of which can be exactly determined. The yellow of sodium—the line marked D in Fig. 25*a* —is proved by a spectroscope possessing a prism or grating of sufficient resolving power to consist of two lines very close together, having the frequencies $508{\cdot}9 \times 10^{12}$ and $509{\cdot}3 \times 10^{12}$ respectively. The sodium atom, as well as the other elements, send out a number of other lines which lie in the ultra-violet region (Fig. 25*a*, main series).

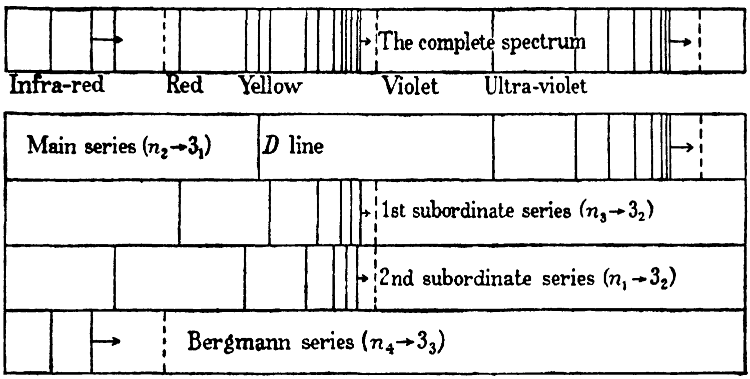


*Fig. 25*a. *Spectral lines of sodium (without the fine structure) separated into series. See pp. 107 and 108 for explanation.*

Substances which are normally gaseous, such as hydrogen or helium, are brought in a rarefied condition into glass tubes, and caused to glow by sending through them a high-tension electric discharge. They then emit their characteristic spectra. These tubes of glowing gases are very familiar to us since their use for advertising signs became common.

Besides the so-called arc spectra which we have just considered, it is possible to cause atoms to emit other lines by exciting them by more powerful methods. We then get what are called spark spectra (Fig. 25*a*). Such spectra result from larger energy changes. Many spectra contain an enormous number of lines. The iron atom for example emits a spectrum consisting of a vast number of single lines, scattered over the whole visible range and also far beyond either end of it. All

these frequencies have been exactly measured. Each of these innumerable light waves must result from the vibration of an electron in the atom. An iron atom must be capable of thousands of vibrations; its range of light-notes is much greater than a piano's compass. The question arises whether it may be possible to deduce from this electrical concert which nature plays on the atom something concerning the structure of the instrument. If only we could draw the right conclusion concerning the mechanism by which this electromagnetic music is produced! We are unable to picture this mechanism in any other way than by analogy with the musical instruments known to us. The spectral lines were discovered by Kirchoff and Bunsen in 1860, and since that time chemists had made use of spectrum analysis for investigating the chemical composition of bodies, while the

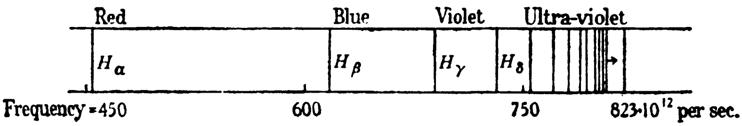


Fig. 25 b. *Spectral lines of hydrogen.*

astronomers had analysed the light sent to us by the stars in order to deduce the nature of the material of which they are composed. Then the chief question became: "What can spectra tell us about the structure of the atom?"

Dictionaries of spectra exist, in which are to be found the electrical melodies played by all kinds of atoms. Long before this question aroused the interest it arouses to-day, when its bearing on the construction of the atom is so evident, Kayser in Bonn had collected, during many years of work, all that was known about spectra. The results were derived from tens of thousands of individual papers. We possessed an exhaustive description of known atomic spectra, but it was merely a collection of facts, the meaning of which was sealed to us. Our knowledge of spectra was thus immense, and yet we were unable to say why an element should emit just those lines known to be characteristic of it.

However, gradual progress was made in the discovery of

regularities hidden in the seeming chaos of these innumerable lines. Bunsen and Kirchoff had already discovered that atoms absorb precisely the same colour of light as that which they emit. A sodium atom is capable of absorbing out of white light falling on it the precise yellow colour which it emits. If therefore we examine by means of a spectroscope white light which has been passed through sodium vapour, we no longer see a continuous spectrum; at one place in the yellow a black absorption line is seen, or two such lines very close together, if the apparatus is powerful enough to separate them. The discoverers of spectrum analysis were thus able to interpret the many dark lines found by Fraunhofer to exist in the spectrum of the sun as absorption lines resulting from the passage of the white light radiated by the sun's solid or liquid interior through an atmosphere of gas surrounding it. Owing to the intense heat, a large number of elements exist in this gas in the form of vapour. These absorption spectra are just as characteristic for each substance as its emission spectrum.

It was also shown that the structure of the spectra of the various chemical elements varies from element to element in a manner corresponding to the chemical properties of each element. The periodic table, in which the elements are arranged in order of increasing weight, shows us that elements having similar chemical properties recur at regular intervals in the series. It is found that these analogous elements have analogous spectra. This characteristic appears particularly clear in connection with the famous discovery made by Zeeman in 1896, that the spectral lines of an atom, when produced in a very strong magnetic field, are split up into several individual lines of varying brightness. Fig. 26 shows us photographs of the so-called normal and anomalous Zeeman effects.

This splitting up of the various spectral lines is different in the case of different elements, and also in different lines of the same element. It gives us a means of arranging the various lines of a given element in series, by forming each series from those lines which are split up in the same manner.

Electric fields are also able to split up the spectral lines, as we see from Fig. 27.

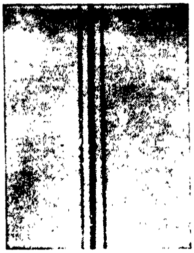


Fig. 26 a. Normal Zeeman effect in hydrogen.

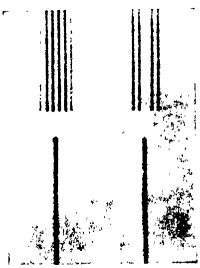


Fig. 26 b. Anomalous Zeeman effect with the yellow sodium line.

[*to face page 78*]

Fig. 27. Splitting up of the lines $H_\delta H_\gamma H_\beta$ *of hydrogen in the electric field according to Stark. The new lines are always divisible into two groups distinguished by the directions of vibration of the light.*

[*to face page 79*]

The Structure of the Hydrogen Atom

The simplest of all spectra is that of hydrogen. Fig. 25*b* on p. 77 shows us on the left the four hydrogen lines lying in the visible parts of the spectrum. They consist of one red, one blue and two violet lines, the distances between them becoming increasingly smaller. This is seen most clearly in the ultra-violet. Here the spectral lines, as we see on the right hand side of the figure, crowd more and more closely together, until they come to a limit. This is not a type of vibration known to us from musical instruments. A flute, for example, which produces a certain frequency when normally blown, is also able to produce, according to the strength with which it is blown, the harmonic overtones, that is to say, the octave of double the frequency, the fifth of three times the frequency, the next octave of four times the frequency, and so on.

Our hydrogen atom does not exhibit this type of regularity in its overtones, but another and exceedingly curious one. Balmer discovered it in 1885. It is as follows: a hydrogen atom is able to emit radiation of all wave-lengths which obey the formula

$$\nu=\frac{R}{2^2}-\frac{R}{n^2},$$

where n may be any whole number from 3 onwards, and different for each spectral line. R is always the same number, that is to say, $3{\cdot}29\times10^{15}$. More exactly, when we use the letter c to denote the velocity of light, which is accurately 299,796 kilometres per second, $R=c\times109677{\cdot}759$. Since we are able to measure the lines of spectra with extraordinary accuracy, this constant is known with corresponding accuracy, that is to say, to nine figures. This is equivalent to being able to measure the distance between London and Brighton to a quarter of an inch. The table of Fig. 28 shows us how accurately Balmer's formula represents the actual measurements. The table gives wave-lengths instead of frequencies.

Still more remarkable is the fact discovered by Rydberg in 1898, that the spectral series of other elements can also be represented by quadratic expressions of a similar type, in which this same number R appears. It is called the Rydberg constant.

Ten years later, Ritz put forward the suggestion that the "terms" of the series might have an independent meaning, and that other terms might be put into the Balmer formula; for example, in place of the first term, another such as $\frac{R}{3^2}$.

Line	n	Wave-length in 10^{-5} cm.	
		Calculated by Balmer	Observed
Red H_α	3	6·56280	6·56280
Blue H_β	4	4·86138	4·86133
Violet H_γ	5	4·34051	4·34047
H_δ	6	4·10178	4·10174
Ultra-violet	7	3·97011	3·97006
	8	3·88909	3·88900
	9	3·83543	3·83538
	10	3·79793	3·79792
	11	3·77067	3·77065
	12	3·75018	3·75018
	. . . .	. . . .	. . . .
	30	3·66229	3·66221
	31	3·66125	3·66121
	. . . .	. . . .	. . . .

Fig. 28. The Balmer series of the hydrogen spectrum. Comparison between values calculated from the formula, and experiment.

The series thus calculated was then searched for by Paschen, who found it experimentally.

On account of the large numerator in the first term, this radiation is in the region of low frequency, in the infra-red. If on the other hand, we put the first term equal to $\frac{R}{1^2}$, the calculation gives us ultra-violet lines, which are actually

found experimentally. The spectrum of hydrogen can be completely described by the formula

$$\nu = \frac{R}{m^2} - \frac{R}{n^2},$$

in which n is any whole number which is larger than the whole number m.

Such was the state of knowledge concerning the spectrum of hydrogen in the year 1913. A large number of measurements of spectral lines had been made, and many relations between them had been found out by trial. But how were these relations to be explained? Chemistry was able to tell us a great deal about atoms, while classical physics had worked out very exact ideas concerning light; but spectra baffled all attempts to explain them. Innumerable observations, expressed in the most exact figures and formulae, such as Balmer's, seemed to be waiting silently but insistently upon the physicist for a solution.

How were we to regard atoms as constituted? Rutherford, as we have learned, maintained that each atom was made up of a nucleus with an excess of positive electric charge, which was compensated by the negative charge of its satellite electrons, which circulated around the nucleus like planets around the sun or the moon around the earth. The question was to discover how these satellite electrons behaved. They can no more be at rest than are the planets in the solar system. A planet at rest would be attracted by the sun, and would plunge into it. An electron at rest would, in the same way, fall into the nucleus, on account of the latter's attraction. The electrons must therefore oscillate or revolve in orbits about the nucleus. The equilibrium cannot be static, it must be dynamic. This conclusion leads at once to a new difficulty. For electrons differ in an important respect from planets. They carry an electric charge. An oscillating electron, or one revolving in an orbit, must radiate electromagnetic waves in rhythm with its movement, just as do the electrons which swing to-and-fro in the antenna of a wireless transmitter. Hence an electron, as long as it is in motion of this kind, must emit light. But atoms do not continually emit light. This

would be impossible, since their electrons would thereby continually lose energy, and would then be attracted closer and closer to the nucleus, and finally rush towards it along a spiral path, in a very short time. Atoms, and hence all matter, could not exist for a second. Classical physics, and Rutherford's atomic model, cannot therefore be reconciled. This atomic model is unable to explain the most fundamental fact of atomic physics, namely the great stability of atoms.

In earlier times, a dilemma of this kind would have led to the atomic model being regarded as wrong. But this model of Rutherford's had been developed on the basis of very accurate radioactive measurements, while classical physics had itself met with insuperable difficulties in the problem of the exchange of energy between atoms. These difficulties had led Einstein to maintain that the classical wave theory must in some way be wrong, and that light is to be imagined as made up of photons. Thereupon, in the year 1913, the Danish physicist and pupil of Rutherford, Niels Bohr, made a bold attempt to combine Einstein's hypothesis of light quanta with the Rutherford model of the atom. It seemed possible that by breaking to some extent with classical physics, a model of the atom could be constructed which would solve the riddle of spectra. A very strong instinct for physical investigation was needed to decide how much of classical physics was to be retained, and to what extent the quantum theory was to be employed.

The law of attraction of electric charges was to be retained in its classical form. This means retaining the result of it, that the electrons move like planets in ellipses or circles, around their sun the nucleus. Bohr first proceeded on the basis of circles. In the planetary system, we have many circular orbits of very different sizes. The radius of a planet's orbit depends solely upon its energy of motion. The greater this energy, the more distant is the path of the planet from the sun. Jupiter for example takes eleven years to travel once around the sun, and is five times as distant from it as the earth. If by any external influence its energy could be increased, it would recede farther from the sun, and travel say in the track of Neptune, which is thirty times farther than the

earth from the sun, and takes more than 164 years to travel once around the sun. The planets with less energy of motion are held by the sun in closer circles, in which they move more rapidly, and are thus prevented from plunging into the sun.

Bohr supposes that the electrons behave in a similar manner in accordance with classical theory. In spite of many speculations by astronomers in the sixteenth and seventeenth centuries, and more recently, no relationships have been found between the various orbits of the planets. All orbits are possible for the simple reason that all values of a planet's energy are also possible. Bohr now leaves classical physics, and maintains that no such state of affairs is true of the atom. The experiments of Franck and Hertz had already shown that an atom cannot possess an unlimited number of different amounts of energy, but that only a few definite quanta $h\nu$ can be absorbed or emitted. Hence, this new quantum condition allows us to state that, of all the paths which classical physics would allow an electron to follow round an atom, only a few are possible. In these paths, also, an electron must be able to travel without radiating. The additional quantum condition which selects these "quantised orbits" must be suitably chosen. If we are to permit the electron to travel only in certain paths separated by definite steps, its velocity, and therefore its impulse and energy, can possess only certain values. We must assume that one of these magnitudes is limited in some definite manner.

General theoretical ideas caused Bohr to choose the impulse. In the case of a ray of light, the impulse is given by the relation $p=\frac{h}{\lambda}$. It is thus determined by Planck's constant h. This new constant of quantum physics which regulates the emission and absorption of light in quanta, might also perhaps characterise the various states of the atom. The wave-length is the distance travelled by light between the points at which it is in the same state of vibration. Now the electron travels in a circular path, returning to the same point each time it performs a journey round the circle. Bohr therefore introduces the fraction $\frac{h}{2\pi r}$, where π is the number

3·1415..., r is the radius of the circle, and $2\pi r$ the circumference of the circle. He assumes that this fraction can only increase by whole numbers. Bohr's "postulate of stationary states" thus runs as follows: the impulse can only assume values given by the formula $p = n \cdot \frac{h}{2\pi r}$, where n is any whole number. All other orbits are ruled out. This seems very arbitrary, and cannot be justified by the principles of classical physics; it is thus unsatisfactory. Let us see to what it leads us. Calculation shows that only such orbits are possible as have radii in the proportion 1:4:9:... that is to say, as the square of the "quantum number n". It also follows that the velocity on the inmost "Bohr orbit" (where $n = 1$) $v_1 = 2187$ km./sec.; it decreases in the outer orbits as the quantum numbers; in the second orbit, it is $\frac{v_1}{2}$, in the third $\frac{v_1}{3}$, in the nth $\frac{v_1}{n}$.

The energy is also "quantised". If we denote by W the energy possessed by a hydrogen atom without its electron, a proton, that is to say, it follows that the energy of the whole atom decreases when it attaches to itself an electron moving in the innermost possible circle. It becomes smaller, let us say, by an amount B; so we may say that $E_1 = W - B$. B thus denotes the energy which must be expended to tear the electron away from the atom. The second possible energy value for the atom is that which it possesses when the electron is travelling in the second Bohr orbit. This energy is given by $E_2 = W - \frac{B}{2^2}$, and is therefore greater, since the original portion of energy is diminished by a less amount. In the third orbit, it is equal to $W - \frac{B}{3^2}$; in the nth, it is equal to $W - \frac{B}{n^2}$. It thus becomes greater and greater as the electron moves out to orbits farther and farther from the nucleus, that is to say, characterised by higher and higher quantum numbers.

The transition from a near orbit to one more distant is governed by Bohr's second postulate, in which he makes use of Einstein's photoelectric equation. Such a transition is

only possible when a light quantum of the value $h\nu$ is absorbed.

In such a case, for example when the electron switches over from the second to the third orbit, the atom gains in energy by an amount given by

$$E_3 - E_2 = h\nu.$$

This is called the "frequency postulate". We have therefore

$$\left(W=\frac{B}{3^2}\right)-\left(W-\frac{B}{2^2}\right)=h\nu,$$

from which it follows that

$$\nu=\frac{1}{2^2}\frac{B}{h}-\frac{1}{3^2}\frac{B}{h}.$$

This is an equation giving us the frequency of the light which a hydrogen atom can absorb when its electron switches over from the second to the third orbit. Conversely: if an electron switches from the more distant third orbit to the second orbit, the atom emits a light quantum of the amount $h=E_3-E_2$. This is the same equation as before. An atom emits the same colour of light as it absorbs. When we put instead of the second and third orbit, quite generally the mth nearer the nucleus, and the nth more distant from it,

$$\nu=\frac{1}{m^2}\frac{B}{h}-\frac{1}{n^2}\frac{B}{h}.$$

This is precisely Balmer's formula, excepting that we now have $\frac{B}{h}$ in place of the Rydberg constant. The question now is, what value do we find by calculation for B? The whole of Bohr's theory would be useless if it were not successful in calculating the correct value for the constant R. We find that

$$B=\frac{2\pi^2 e^4 m}{h^2},$$

where e and m denote the charge and mass of the electron. This gives us for $\frac{B}{h}$ the value $3{\cdot}27 \times 10^{15}$, that is to say, exactly the Rydberg constant.

In other words, Bohr succeeded, by means of these two daring quantum postulates, in calculating the spectrum of hydrogen. How accurately this was done, we see from Fig. 28 (p. 80). This was a success denied to classical physics, which knew nothing of Planck's constant. Bohr's calculation

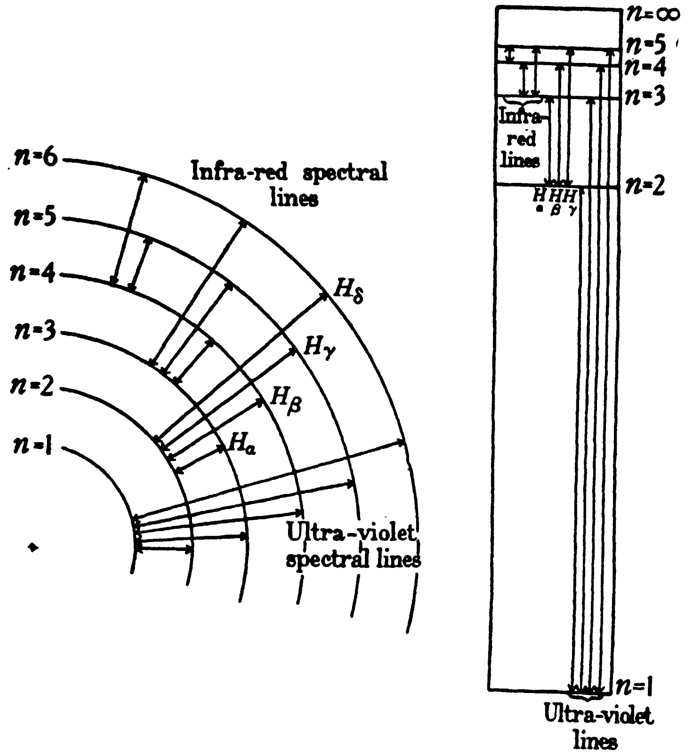


Fig. 29. The spectral lines of hydrogen are represented as "switches" from one circular orbit to another. On the left-hand side the orbits are shown diagrammatically, on the right the energy levels are represented correctly to scale.

showed this constant to be fundamental to a comprehension of the structure of atoms.

Let us once more envisage clearly what we see when we examine the hydrogen spectrum (Fig. 29). We know that the visible lines H_α to H_δ appear when $m = 2$ and $n = 3$, 4, and 5. These correspond to switches of electrons from the third,

fourth and fifth orbit to the second. When an electric discharge is passed through a glass tube containing hydrogen, we must imagine the following process taking place. The addition of energy to the gas results in numerous atoms of hydrogen, among the many trillions present, having their electrons switched from orbits close to the nuclei to more distant orbits. These electrons strive to jump back into the more stable orbits. When they do so, the energy which they have taken up is radiated as light of a single wave-length. Many millions of electrons will jump from the third orbit to the second, many others from the fourth to the second, and so on from the fifth, sixth, seventh to the second, and also to the first and third. Hence all spectral lines appear at the same time. If, on the other hand, we send through the gas, in which the electrons have fallen back into the more stable orbits near the nucleus, light quanta of the value $h\nu$, and choose the frequency γ of this light suitably, the energy may be absorbed again, and the electrons thrown back to the more distant orbit, which they left in radiating light of this frequency. The absorption lines are in precisely the same position in the spectrum as the emission lines. This is in full agreement with the experiments of Franck and Hertz. Not only hydrogen atoms, but all atoms, are unable to absorb light of any but certain colours. The frequency γ of the photons must be exactly such that their energy $h\nu$ is equal to the difference in energy between two possible orbits:

$$h\nu = E_m - E_n .$$

For only certain definite orbits are "permitted" to the electron.

Bohr thus actually succeeded, by applying limiting quantum conditions of a completely extraneous character to classical physics, by postulating the existence of stationary orbits in the atoms, and by his frequency postulate, in explaining the spectral lines emitted or absorbed by hydrogen. It was particularly remarkable that, in complete contradiction to classical physics, the wave-length of the radiated light was not determined by the periodicity of the electron's movement, either in its initial or its final orbit. The frequency

depends, on the contrary, only upon the energy difference between the two paths. However, when the switch takes place between paths very far from the nucleus, say the hundredth and the hundred and first, the energy difference becomes very small. We are then dealing with the minute light quanta of the infra-red, and the possible energy values pass almost continuously one into the other. In this region of long wave-length, the quantum theory tells us the same story as the classical theory. Classical physics thus has the character of a good approximation, made by thinking in terms of continuity about reality, which in its essential nature is not continuous, but operates in jumps. In this region, calculation shows that the values deduced for the frequencies by the two theories agree. The frequency deduced from the rotation of the electron should, according to the classical theory, be equal to that of the radiated light; in these distant orbits, it is the same as that calculated from the quantum switch of an electron from one orbit to the next.

This was our first and very strange glimpse behind the mystery of spectral lines, and it produced an enormous impression upon the world of physics. It exposed, even more plainly than the numerous experiments which we considered in the last chapter, the inadequacy of classical physics as applied to atomic dimensions; the supreme importance of the constant h became clear. Classical physics could only be applied to the interior of the atom with great limitations, and these were completely irreconcilable with its fundamental principles. The light which is emitted or absorbed by an atom appears not as a wave, but as a corpuscle, and the atoms are to be regarded as planetary systems in which quite incomprehensible prohibitions as to orbits exist. Planck said once, concerning Bohr's theory: "It is a theory, the magnificent successes of which contrast with the boldness of its assumptions, and with the completeness of its breach with long accepted, well founded and thoroughly tested views, in a way which has no precedent in the history of exact science."

This impression was further strengthened when physicists proceeded to work further with the Bohr atomic model, and found that it was capable of doing a great deal more than

Bohr had shown it to do. The starting point was the fact that a very powerful spectroscope is able to break up the hydrogen lines and prove that they are not simple, but consist of several lines lying close together. They possess a "fine structure".

More lines denote that there are more possibilities for the electron to switch over than are given by the Bohr circles. The obvious step was to follow the astronomical analogy, and assume that the orbits need not be circles, but may be ellipses, which may occasionally be reduced to circles. Sommerfeld attacked this problem. When he applied Bohr's method of calculation to ellipses, he actually obtained the required larger number of orbits. The calculation shows that

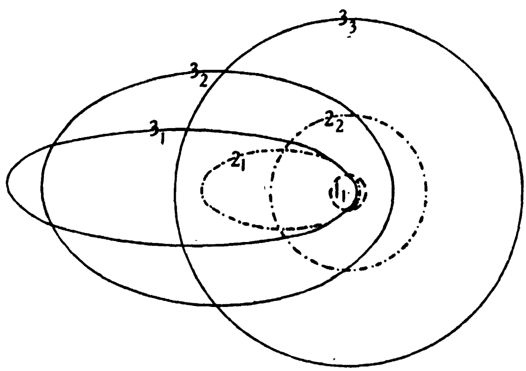


Fig. 30. First, second and third elliptical orbits according to Sommerfeld.

ellipses must be substituted for circles. The innermost single quantum orbit is again a circle. The two-quantum orbit with fourfold radius might be a circle or an ellipse with a major axis of the same length. In the place of the three-quantum circles, we had a circle and two ellipses, all again having equal major axes; with each new quantum number we get a new ellipse. All these ellipses have a focus in common, at which we are to imagine the nucleus to be situated. Each orbit is then characterised by two quantum numbers, as we see from Fig. 30.

The major axes increase from group to group, according to the square of the principal quantum number n. The ellipses of the same principal quantum number are distinguished by

a new, second quantum number, the "azimuthal" quantum number k. This measures the velocity, varying in steps from orbit to orbit, with which the surface is swept by the radius vector (Fig. 31). It is also a measure of the length of the minor axis of the ellipse to which it belongs. For example, the principal quantum number 3 is possessed by three ellipses including the circle 3_1, 3_2, 3_3 with the values $k=1$, $k=2$, $k=3$. These ellipses have minor axes of lengths 1/3, 2/3, and 3/3 respectively of their major axes (the last being of course a circle); the two-quantum orbits consist of an ellipse 2_1, the minor axis of which is 1/2 the major axis, and a circle 2_2, with equal major and minor axes. In general, every principal quantum

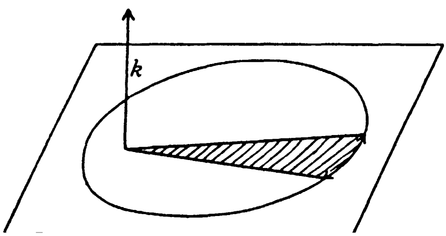


Fig. 31. The azimuthal quantum number K *measures by its length the velocity with which the radius vector sweeps over the shaded circle. It is drawn perpendicular to the plane of the ellipse.*

number n comprises as many ellipses as are given by azimuthal quantum numbers $k=1$, $k=2$, ... up to $k=n$. Sommerfeld thus calculated the increased number of the orbits, but at first a disappointment turned up, for the calculation showed that the energy of the electron, in every orbit, depends only upon the major axis. But this is the same for all ellipses possessing the same principal quantum number. Hence the difference of energy is just the same, whatever the ellipse to which the electron jumps, so long as it possesses the same quantum number. This makes the frequency of the spectral lines emitted the same, whereas they should be slightly different.

Sommerfeld thereupon introduced a refinement into his calculation. The electrons are moving, as we have seen, at a considerable speed. This means that the old-fashioned

mechanics cannot hold good, but that the theory of relativity must be applied. In this, the mass of a body is not constant, but varies with its velocity. Since the velocity in the various elliptical paths belonging to any one quantum number is somewhat different, the masses of the electrons must also differ slightly, and hence their energy must also be different, according to the well-known equation $E=mc^2$. This means that there must be a difference in the amount of energy radiated, according as the electron switches to one or the other of the elliptical orbits. Thus all energy terms, and hence the spectral lines, are complex. They possess a fine structure of several values lying very close together. The elliptical paths themselves are also modified. Close to the nucleus, the electron moves faster in its track than at the points where it is farther away. Hence its mass in "perihelion" is slightly different from what it is in "aphelion". The ellipse is therefore not equally curved at these two points, and hence does not return upon itself, but rotates somewhat as a whole. It thus has a precessional movement, similar to, but much smaller than that required by the general theory of relativity (Fig. 2, p. 9) which, as we saw, requires such a movement in the presence of strong gravitational fields.

Sommerfeld was led by these more exact considerations to a new formula, which predicted a perfectly definite fine structure for the spectral lines of hydrogen. This fine structure was examined quantitatively by Paschen. Without experimental confirmation, this beautiful theory would have remained a beautiful dream. The measurements are extraordinarily difficult, for the individual lines of the fine structure lie quite close together. Besides hydrogen, Paschen examined the spectrum of ionised helium, that is to say, of atoms consisting of the helium nucleus which has lost one of its two external electrons. This case had also been calculated by Sommerfeld. The theory was found to give the actual fine division of the lines with the most astonishing accuracy. The impression was quite general that both the relativity theory and the quantum theory had revealed their physical reality in this application to the behaviour of electrons. Planck said concerning Sommerfeld's work: "This represents a feat fully

on a level with the famous discovery of the planet Neptune, the existence and orbit of which were calculated by Leverrier, before it had been seen by any human eye." At first science had trodden only very cautiously in the path pointed out by Bohr, for his two quantum conditions seemed to represent a violent and incomprehensible breach with traditional and classical physics. After this triumph of Sommerfeld, physicists were much more disposed to renounce their objections, and take full advantage of the eminent utility of the new theory. It appeared later that the explanation of the fine structure given by Sommerfeld was in reality no more than a very remarkable accident. There is another cause operative, as Dirac pointed out, namely "electron spin", with which we shall later become acquainted. Fortunately, it makes no difference to the number and position of the lines.

Other physicists also followed the lead given by Bohr. At about the same time as Sommerfeld's discovery, Schwarzschild and Epstein succeeded in giving a complete explanation of the fact discovered three years previously by Stark, that the spectral lines of hydrogen are split up when the atoms emitting them are in a strong electric field (Fig. 27, opposite p. 79). This theory of Bohr, in spite of its being a mixture of classical and quantum points of view, was found to fit the facts to an astonishing degree. This astonishment was expressed in the words used by Epstein in his first communication: "We believe that our results form a new and striking proof of the correctness of the Bohr atomic model, which cannot fail to carry conviction even to the most reserved of our colleagues. It seems that the power of the quantum theory, as applied to this model, is truly remarkable, and very far from exhausted."

We have already referred to the famous parallel case, the Zeeman effect. The famous Dutch physicist, H. A. Lorentz, had already predicted in 1895, on the basis of his theory of electrons oscillating in atoms, which theory was the final outcome of the Maxwell theory of electricity, that spectral lines would be split up in a magnetic field, and how they would be split up. In the following year, Zeeman succeeded in demonstrating this separation of the lines in a strong magnetic field, and showed that it agreed exactly with

Lorentz's prediction, at any rate in the simplest, or "normal" case. Such a case is presented by hydrogen, particularly when the measurement is not performed with too great refinement (Fig. 26, opposite p. 78).

The effect of a magnetic or electric field upon an electron moving in an elliptical orbit according to Bohr is to cause

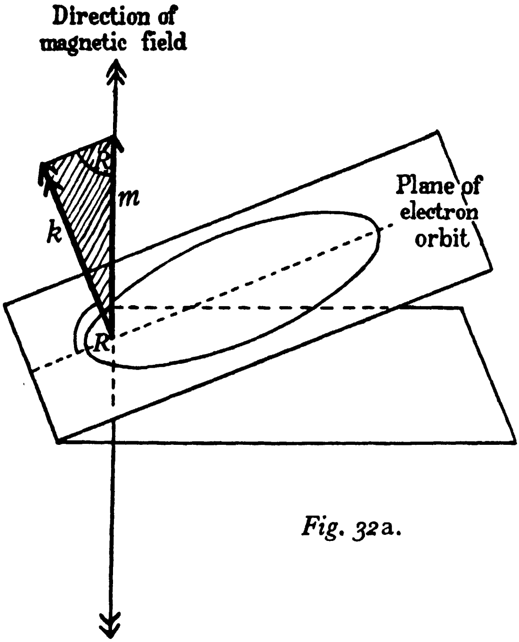


Fig. 32a.

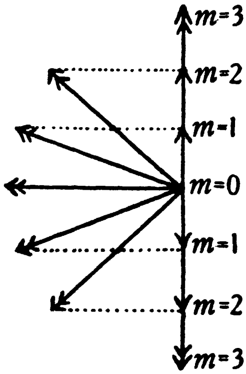


Fig. 32b.

Fig. 32a. *Space quantisation.*

Fig. 32b. *The arrows show the possible directions for* k = 3. *The planes of the ellipses are in each case at right angles to them. The corresponding values of the number* m *are indicated on the normal which gives the direction of the magnetic field.*

the axis of the ellipse to rotate slowly about the direction of the magnetic field, at a speed which is greater, the greater the strength of the field. It furthermore appears that the plane of the ellipse, concerning the position of which it has hitherto been unnecessary to make any assumptions, can only take up certain few definite positions, as soon as an external magnetic field is applied, no matter what its

strength may be. This is called the space quantisation. The possible "permitted" positions for the ellipse are given by drawing in Fig. 32 from the end point of the arrow the length of which denotes the azimuthal quantum number k, a perpendicular to meet the direction of the magnetic field. *We then get in this direction a new arrow of length m.* The space quantisation consists in the fact that this arrow itself may only be a whole number in length; for example, when $k=3$, it can only have the seven values, $+3$, $+2$, $+1$, 0, -1, -2, -3, in which $+$ and $-$ denote the opposite directions of space. We thus have altogether $2 \times 3+1=7$ in general $2k+1$ directions for the arrow marked with a k, and hence the same number for the ellipse. They are distinguished from one another by a new, third, magnetic number m, which can have all whole-number values from $-k$ to $+k$.

This conception of space quantisation is found in practice to give a completely correct calculation of the normal Zeeman effect. Space quantisation itself was quite independently proven in the year 1921 and subsequently by Stern and Gerlach. The only doubtful point is, that if we study it exactly, the magnetic separation of even the hydrogen lines is complicated and anomalous (Fig. 26, opposite p. 78), in a way frequently found in the case of atoms possessing more than one electron. This fact cannot be explained without fresh assumptions.

A further objection may be raised to the Bohr-Sommerfeld model of the hydrogen atom. The model makes it out to be flat, whereas all experiments on hydrogen lead us to expect its atom to have the shape of a sphere.

Apart from these two objections, the fundamental disharmony in principle remains, although this was temporarily set aside when the Bohr theory proved triumphantly successful. The laws of classical physics are used to determine the path of the electron, but their validity is denied immediately afterwards. For their further results are negated by two quantum postulates, which are quite irreconcilable with classical theory. It is impossible to find logical grounds for introducing these postulates into the standard system of physics. But if this cannot be done, the whole Bohr theory, in

spite of its success, can only be regarded as provisional. We could quote many expressions of doubt by leading physicists uttered during the years of uncertainty, from 1916 to 1925. But nevertheless, the great successes obtained repeatedly fascinated the scientific world, and many felt confident that the difficulties of principle would somehow be overcome. They seemed but a small blemish on the beauty of the new theory.

Schrödinger once said at a later time: "The situation was rather desperate. If the old science of mechanics had failed completely, we might have let it go. For then the way would have been open to work out an alternative on new lines. But as it was, we were presented with the difficult task of preserving its spirit, which we clearly felt to pervade our Universe, while at the same time placating it to the point of accepting the quantum postulates, not as riding roughshod over the theory, but as in some way akin to, and proceeding from, its inmost nature."

But hydrogen is only one, and furthermore the simplest, atom which emits spectral lines. The unsolved riddle might perhaps be read if quantum physics were to proceed to undertake the interpretation of the enormous material presented by the spectra of the other chemical atoms. We might then be led to see at what point our previous ideas are in need of correction.

CHAPTER V

The Higher Atoms

Although the pursuit of natural science begins by stripping the world of its magic, the shock we receive from the new and strange, when we reach the limits of our knowledge, brings us face to face with the unknown in a new and direct form.

—KARL JASPERS, Philosophische Weltorientierung.

The results we have just described were remarkable and surprising. Planck's hypothesis that there is something discontinuous in the process of nature, and that the constant *h* is characteristic for these quantum phenomena, had found, as we have seen, remarkable support from the success obtained by Bohr and his followers in reaching an understanding of the spectral lines emitted by the hydrogen atom. New and attractive problems arose. The spectra of the other ninety-one elements had to be read, as well as that of hydrogen; for insight into the structure of these more complicated microcosms might lead to the discovery of the nature of chemical forces, and so to an understanding of chemical processes. We might also find a broader base for the quantum principles of the Bohr model, and be able to verify them to a much greater extent; we might thus be led to a solution of the difficulties which had hitherto proved insoluble. In the decade following Bohr's first forward step, this problem was attacked on all sides with the greatest energy. If we look at the physical journals of this period, it seems as if hardly any other physical problem existed besides that of the structure of atoms with several electrons.

But the spectra are only one, though a particularly important and well studied, property of chemical elements. Chemistry had discovered a large number of other regular relationships between them. We must now go into this matter, as far as it is necessary to the understanding of our theme, the structure of the atom.

In the year 1869, two men* independently discovered the fact that the chemical elements can be arranged in an order which obviously has a relation to their properties. These pioneers were the German Lothar Meyer, and the Russian Mendeléeff (Fig. 3, pp. 16, 17). They arranged the elements in the order of increasing atomic weight, from hydrogen of atomic weight 1·008 to uranium of the atomic weight 238·14, and discovered that they then fall into successive series of eight elements, in such a way that elements having the same position in each series are similar in chemical properties. One of the most important chemical properties changes periodically and suddenly, namely the valency, that is to say, the number which states how many atoms of hydrogen will combine with a single atom of the element in question, or can be displaced by such an atom from a chemical compound. In the first column of the table we find the lively "monovalent" elements known as alkali metals, lithium, sodium, potassium, rubidium, and caesium, together with the metals copper, silver, and gold, which are also in part monovalent. In the second column we find the somewhat less active "divalent" alkaline earth metals, calcium, strontium, and barium, together with some divalent heavy metals, zinc, cadmium, and mercury. In the next column the valency rises to 3 for the "earths", to which aluminium belongs. Next we have the first group of non-metallic elements, the "tetravalent" carbon and silicon, which, with their enormous number of compounds, enter into the composition of by far the great majority of all substances both in the living and the non-living world. The next group, nitrogen and its analogues, is "pentavalent", then comes the "hexavalent" oxygen family; next, the elements most strongly non-metallic, the halogens fluorine, chlorine, bromine, and iodine. We will presently deal with some further peculiarities.

* Mention should be made of de Chaucourtois, a French chemist, and Newlands, an Englishman, who, independently of one another, discovered the principle; de Chaucourtois' first paper was published in 1862, that of Newlands in 1864. Mendeléeff's work was done in ignorance of theirs, and he was more fully aware of its implications, in particular the presence of gaps representing undiscovered elements. [Trans.]

The valency of an element is of prime chemical importance, and this periodic, whole-number change in it with change of atomic weight shows that the chemical properties depend upon the latter, in spite of many particular problems; similar properties recur periodically in the series. By way of transition between the halogen group with its strongly non-metallic character, and the metals, we have elements of zero valency. These were discovered much later, and are incapable of entering into chemical combination. They comprise the "noble" gases helium, neon, argon, krypton, xenon and radon.

The original discovery of this law was thus of the greatest value. It was also destined to bear immediate fruit. Certain elements, such as beryllium and indium, did not fit into the system. Mendeléeff made use of it to predict that their true atomic weights as then accepted would be found to be false, and this was afterwards confirmed by new determinations, which gave values close to Mendeléeff's predictions. But not only were these and other elements put in their proper places. Mendeléeff was able to foresee the discovery, and also the properties, of new elements needed to fit gaps in the table. A few years later, some of these elements were actually discovered, and their properties were found to be those predicted by Mendeléeff.

In recent times, nearly all the elements still missing have been discovered, and found to fit into the remaining gaps. These recent elements have mostly been given patriotic names. Besides those predicted by Mendeléeff, namely gallium, germanium, and scandium, we now have polonium, discovered by Madame Curie, and quite recently hafnium, rhenium, masurium, and illinium. Only two gaps remain in the table; a halogen and an alkali metal are still waiting to be discovered.

It would thus seem as if we were here on the track of the principle according to which the chemical elements are constructed. The case recalls the biblical story of Creation. There we have from evening and morning the first day, here we have from the eight elements from alkali metal to noble gas the first period. And then the process is repeated with a

further eight elements to the next noble gas. We might almost repeat the biblical verdict: "and God saw that it was good". But this would be a hasty verdict, for we do not yet understand the system sufficiently to form a judgment. There are all kinds of irregularities, which we have so far neglected. The periods are not all periods of eight. The first period comprises only two elements, hydrogen and helium. After two correct periods of eight, we come to two long periods, the elements of which can only be arranged under one another somewhat arbitrarily. The elements in the first of the two lines agree fairly well with the remaining elements in the vertical columns, but only in certain properties: the iron and the platinum groups have to be taken out altogether and put into an eighth column, three at a time, for otherwise the whole system would fall into confusion. After these two long periods of 18 elements each, we have a very long one of 32 elements. Here occurs also the group of the "rare earths", 14 in number, for which there is really no place in the system. They are, as it were, put into an annexe by themselves, outside the main building. But a certain mysterious regularity is visible behind these irregularities, as for example the number of elements in each period: 2, 8, 18, 32. Rydberg pointed out that these numbers are the doubles of squares, thus:

$$2 \times 1^2 = 2 \quad 2 \times 2^2 = 8 \quad 2 \times 3^2 = 18 \quad 2 \times 4^2 = 32.$$

But why these doubled squares should turn up is still a mystery. The whole table resembles an extremely difficult crossword puzzle. Since it is set to us by nature, and not by man, it is by no means easy to solve. Twenty years ago we were still baffled. But the time came when we were able to raise the curtain a little, and, by the aid of the quantum theory, to see a little more deeply into the matter. The riddle is no longer merely a curiosity, but charged with the profoundest significance. Through it we are brought face to face with new problems of which we never dreamed.

At one time it was believed that the irregularities merely depended upon an unskilful arrangement. The discoverers of the law had already employed, in addition to the short period form which we have given, the long period form

embodying 18 elements. But this still fails to make room for the rare earths. Also, all sorts of other arrangements were tried, for instance, spirals or screw-shaped lines in space. But all these attempts failed. Meyer and Mendeléeff were not in possession of the necessary facts. New facts had to be discovered. Then only could one proceed to consider answering the following interesting questions. Whence come the curious double-square numbers? Whence the group of 14 rare earth elements of almost identical chemical properties? How is the whole-number change in valency, and its periodic recurrence to be explained? Or when we consider the atomic weight: why have so many elements whole-number atomic weights? Or the same question in another form: why are they not all whole numbers? Are they not all, as Prout maintained in 1815, multiples of a single small number, so that all elements are built up from a different number of sub-atoms of one original substance? Why is the fundamental principle of the system completely violated at four separate points? In order to prevent completely different elements coming under one another, it was necessary to put argon in front of potassium, tellurium in front of iodine, cobalt in front of nickel, and thorium in front of protactinium, instead of in each case the opposite order. Perhaps after all, the atomic weight is not the true key to the right order, but some other number? In fact, this has turned out to be the case.

In the first place, the heavy radioactive elements were found to be mixtures of elements differing in atomic weight, but otherwise possessing completely identical physical and chemical properties, and hence inseparable by ordinary methods. We could measure only the average atomic weight of this mixture of "isotopes", as such chemically identical atoms of different weight are called. This average need not be a whole number, even if the weights of the separate isotopes were whole numbers. For this reason, the atomic weight of thorium is greater than that of protactinium, for it contains more isotopes of high atomic weight. The same was then shown to be true for non-radioactive elements, by Aston's wonderful experiments begun in 1920. He produced a beam of atoms, electrically charged and moving at a high

speed, and subjected it to the combined action of an electric and a magnetic field; these deflected atoms differing in weight to a different degree from their paths. Under the combined action of the fields, atoms of the same weight and charge were brought to a focus on a photographic plate. He was thus able to measure the atomic weight of these atoms with the greatest possible accuracy; he found these weights to be whole numbers. In spite of their difference in atomic weight, isotopes of the same element belong in the same position in the periodic system, for their other chemical and physical properties are identical. They are therefore given the same atomic number. Most elements have turned out to be mixtures of isotopes of different weights. American physicists have recently shown that ordinary hydrogen consists of a mixture of atoms of weight 1 with two isotopes of weights 2 and 3. These isotopes are present in very small amount in ordinary hydrogen; only the weight 2 isotope has yet been isolated in quantity. This "heavy hydrogen", or rather the "heavy water" containing it, is now an article of commerce. It is prepared quite easily by electrolysing dilute sulphuric acid; nearly all the hydrogen given off is the light isotope, while the heavy isotope remains behind. This discovery is of the greatest importance to chemistry, for reasons which cannot be discussed here.

The physical meaning of the atomic number was discovered by Rutherford and his pupils, as we have seen in the first chapter. Radioactive measurements have shown that the atomic number is equal to the number of electrons surrounding the nucleus. This number of electrons, or the equivalent number of excess positive charges possessed by the nucleus, is thus the true basis of the periodic system. The properties of the elements change periodically with the number of their electrons. This received an impressive illustration, when the English physicist Moseley succeeded, in 1915, in causing the various elements to emit X-rays of characteristic wave-length by bombarding them with cathode rays. The wave-length of the X-ray spectrum of each element is measured by a crystal grating. He discovered that the frequencies of the X-ray lines do not follow the periodicity of the system, but increase

uniformly right the way through it. The square root of the frequency increases proportionately to the atomic number.

If, as in Fig. 33, we plot the frequency, or the square root of it as in Fig. 34, against the atomic number, we get a parabola or straight line respectively. These figures also allow us to check the atomic number; they point out gaps, such as,

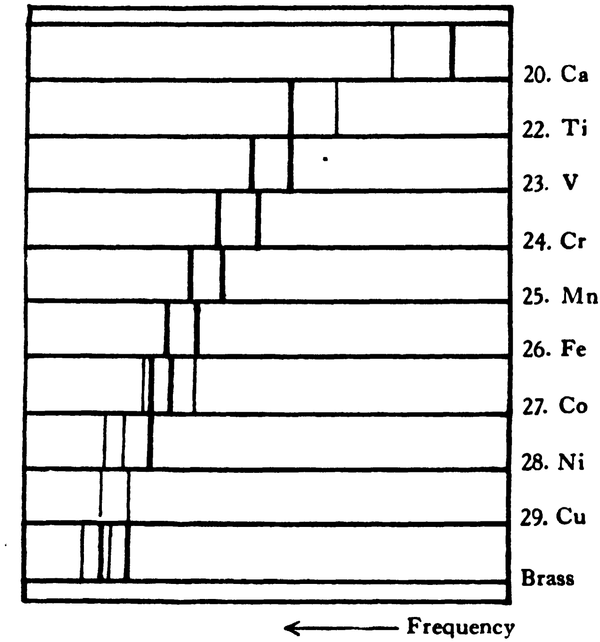


Fig. 33. The X-ray spectra (K series) of the elements no. 20 Ca (no. 21 Sc omitted), no. 22 Ti, to no. 29 Cu, according to Moseley. Brass, an alloy of copper and zinc, shows the lines of both elements. Lines of like frequency are set under one another. The frequency increases from right to left.

for example, the missing element no. 21 Sc in Fig. 33, and the missing element no. 43 (masurium) in Fig. 34. They also allow us to determine the number of elements, namely 92. They facilitated the discovery of new elements, in particular hafnium, masurium, and rhenium, for the frequencies of their X-ray spectra were known beforehand. By dropping the original assumption that the atomic weight is the most funda-

mental characteristic of the atom, and by realising that the atomic number, that is the number of its elementary charges, is its truest characteristic, we find ourselves in a much better position to investigate the physical and chemical behaviour of the elements. Their most essential feature is this set of numbers 1–92.

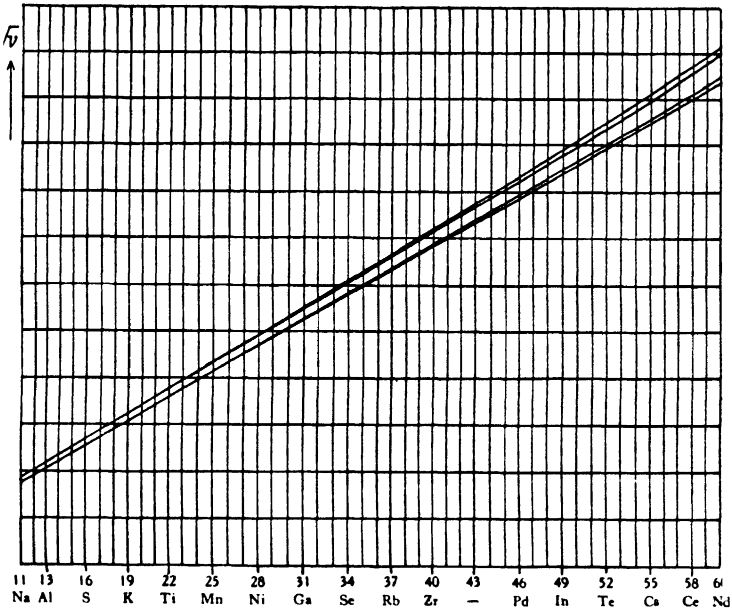


Fig. 34. The square roots of the frequencies of the K series of X-ray spectra of elements no. 11 Na to no. 60 Nd, plotted against their atomic numbers.

Along with many other physical properties, such as compressibility, expansion coefficient, melting point, electrical conductivity, the character of the visible spectrum also changes periodically from element to element. The fact already mentioned that the yellow line of sodium is a doublet recurs in a similar form in the case of chemically similar elements. There are doublets, triplets, and so on as far as octets, and this multiplicity of the spectral lines also exhibits a certain periodicity, which however we cannot further discuss.

The frequencies of the spectral lines of all other elements can also be represented as the differences of two "terms"; the formulae contain the Rydberg number, and also a succession of whole numbers, the principal quantum numbers n. Thus the spectral lines of the other elements result, like those of hydrogen, from the energy switches of electrons between stable quantum orbits. Bohr's frequency postulate must hold quite generally. The frequency ν of the lines tells us the amount of energy $h\nu$ emitted or absorbed. The principle has also proved true, that by combination of any two terms, we again get a line characteristic of the element. However, not all combinations of two terms actually give us observable lines (the same is true of the fine structure of hydrogen). For the intensity of many lines is vanishingly small, unless quite unusual means are used to excite them. Experience enables us to state rules according to which certain energy changes are to be regarded as "forbidden". The Bohr theory gives us grounds for an understanding of these rules, but is not able to supply a strict proof of them.

The unravelling of the laws of spectra has been a gradual process. For many spectra consist of an enormous number of lines, and it is difficult to discover how to group these in series. Various theoretical quantum ideas first put forward by Bohr, and then by a number of others, together with purely empirical rules, have greatly contributed to the investigation of spectra during the last ten years. Sometimes, theory provided a spur to new experimental work; at other times, empirical discoveries led to a revision of theoretical ideas. Interesting as the story is, it would take us too long to follow it up; we must also pass over the method by which the terms have been deduced from the complicated mass of lines, which of course result from differences of terms. Quantitative calculations are very difficult; for in the case of all atoms with more than one electron, we are dealing with the mutual influence of at least three bodies. The mathematical problem can only be solved approximately, as we know from astronomy, where it appears in calculation of the orbits of the planets. We will attempt, following Bohr's hydrogen model as closely as possible, to construct a model for other atoms, with

the object of giving, in the first place, a qualitative explanation of spectroscopic observations.

Moseley's law of X-ray spectra shows that with each new element, the atomic number increases by 1; that is to say, a new electron is acquired by the atom. The chemical valency also increases from element to element by 1; but from time to time a particularly stable configuration is reached, and we have a noble gas. Such a gas possesses no chemical activity, and cannot form chemical compounds; its valency is thus 0. The electrons which it has acquired will be grouped sym-

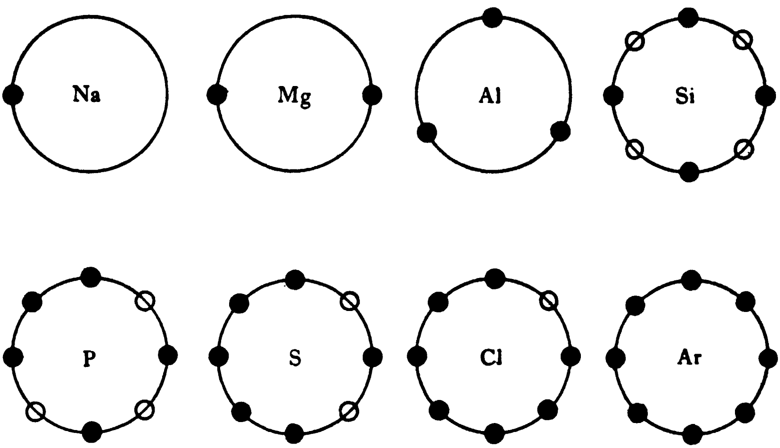


Fig. 35. The outermost electron shells of the elements sodium to argon according to Kossel. ● *=Electrons which can be given up.* ○ *=places which can be filled with electrons.*

metrically around the nucleus. With the next element the valency again increases from 0 to 1, that of an alkali metal; and so on up the series. The new electrons arrange themselves in a new "shell". Let us start, for example, from a noble gas such as neon. In accordance with its atomic number, it possesses ten electrons (Fig. 3, pp. 16, 17). The next following element is the monovalent alkali metal sodium which acquires a further, eleventh electron (Fig. 35). The following element, bivalent magnesium, has two electrons in the outermost shell. In the elements aluminium, silicon, phosphorus, sulphur, chlorine, argon, the number of electrons increases,

step by step with the valency, from three to eight, over and above those possessed by neon. This brings us to a new noble gas argon, which again is surrounded by a complete outer eight-electron shell, indifferent to all external influences. The valency is thus determined by the number of electrons in the outermost shell, incomplete except in the case of the noble gases. Most chemical and physical properties, and also the energy levels revealed in the spectra, which also change periodically with the valency, are determined only by the electrons of the outermost shell, the "valency" or "optical" electrons.

The simplest relationships are therefore to be expected in the case of the alkali metals with their single valency electron. Let us first consider this simple type of higher atom. In this case, we may expect as a first approximation "hydrogen-like" properties. Here we have a single valency electron rotating around an atomic core, that is to say a nucleus surrounded by complete electron shells. We thus obtain an "optical electron" model. The single electron, with a given principal quantum number n, is able, like the hydrogen electron, to take various orbits, namely all n ellipses which are given by the various values of the azimuthal quantum number k. These orbits all exhibit, in the case of hydrogen, a small rotation of perihelion, different for the different orbits (Fig. 2, p. 9). This was caused by the fact that the energy of the electron in the various orbits was somewhat different. In the case of our atom, the optical or valency electron will influence the complex core, made up of nucleus and complete shells, around which it is rotating. It will repel the electrons in it which have a like charge to itself, and attract the nucleus, which is oppositely charged. This will result in its being more strongly attracted when in perihelion, nearest the nucleus. Its path will thereby be more strongly curved. This means that the orbit will rotate much faster than in the case of the hydrogen electron, and this rotation will be very much greater when the ellipse is so eccentric as to partially penetrate the complete shells of electrons (Fig. 36). With this more rapid rotation, the amount of energy in the various paths having the same principal quantum number will now be

considerably different, and not, as in the case of hydrogen, only very slightly different. This explains the fact, long known experimentally, that the alkali metals possess not one, but several, series of spectral lines. The orbits belonging to the same principal quantum number, but now differing considerably in their energy values and possessing the azimuthal quantum numbers $k = 1, 2 \dots n$ belong each to a different set of energy levels. The energy levels 1_1, 2_1, 2_2, 3_1, 3_2, 3_3, and so on, each characterised by two quantum numbers, n_k, may be arranged as in Fig. 37; the levels of smallest energy are lowest, while the steps by which the electron can be raised in

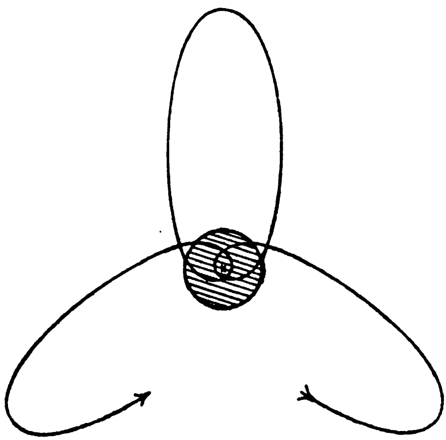


Fig. 36. Elliptical orbit which enters the core of the atom (Sommerfeld).

level by absorption of energy, hence the orbits farthest from the nucleus, lie higher and higher.

In actual fact, the term arrangement of sodium as found experimentally agrees exactly with the pattern expected (Fig. 38, to be compared with Fig. 25*a*, p. 76). Terms with principal quantum numbers $n = 1$ and $n = 2$ do not exist, the two innermost shells are completely occupied with electrons in the case of the third alkali metal; while for every other principal quantum number we have several corresponding levels, three for $n = 3$, four for $n = 4$. The energy differences between terms of the same principal quantum number are actually considerable. The terms with $k = 2$, not to say $k = 1$,

lie considerably lower than the corresponding values for hydrogen. They correspond to very elongated ellipses. In the case of the alkali metals, the model gives at least qualitative, and approximately quantitative, predictions concerning the positions of the energy levels. The lines of the spectrum are produced by the switch of the valency electron from one level to another. The arrows in the figure show the corresponding amounts of energy.

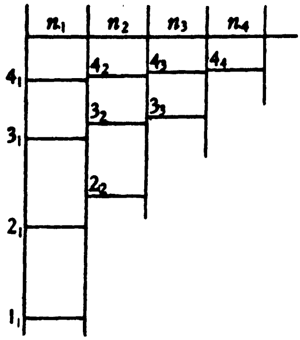


Fig. 37. The energy terms of an element characterised by the principal quantum number n, *and the azimuthal quantum numbers* k = *1, 2, ...* n.

We see that the electron cannot switch to any level that it likes. Here also we have a selection rule, which is the same as that peculiar to hydrogen; the azimuthal quantum number k must increase or decrease by 1, when the electron changes its level. Thus, for example, there is a series of lines which ends in the lowest, and therefore most stable, level, the principal series; that is to say, they arise from the switches

$$3_2 \to 3_1;\ 4_2 \to 3_1;\ 5_2 \to 3_1.\ \text{In general, } n_2 \to 3_1.$$

The first of the spectrum lines given is the well-known yellow *D* line. The lines of other series can be recognised by comparison with Figs. 25*a* (p. 76) and 38.

One point is not explained by this valency electron model. It is the fact that the yellow line of sodium, when carefully examined, is a doublet, as it is called; the same is also true of other lines. The actual experimental facts can only be correctly reproduced when all energy levels are regarded as leading to doublets lying very close together. Only the levels in the first column, with the lowest value of k, are not doublets, but "singlets". Our electron model with two quantum numbers is evidently inadequate to explain this more complicated set of facts. It must be improved.

At first it was believed that the double levels might be explained by assuming that the core of the atom, which is

much larger than the hydrogen nucleus by reason of its completed shells of electrons, rotates around one of its own axes. According as the rotation is in one direction or the other, we have a somewhat different energy state. But this assumption had to be given up. The core has the set of completed electron shells peculiar to the noble gas preceding the element in the periodic system. In the case of sodium, this

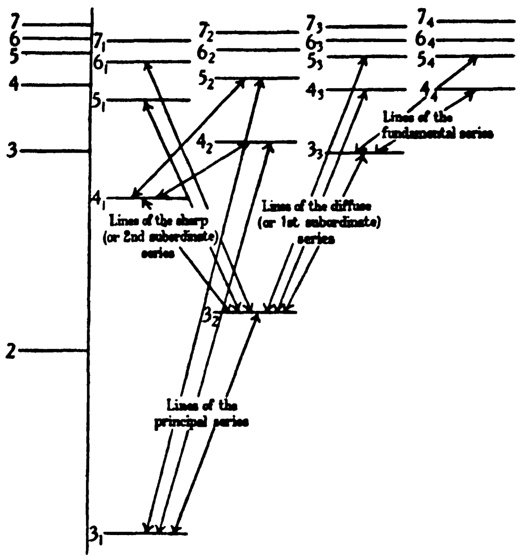


Fig. 38. Energy levels of sodium. Four sets. The set of hydrogen levels shown on the left for comparison enables us to see how greatly the energy in the eccentric orbits (k = *1 or 2*) *of sodium differs from the corresponding values for hydrogen. The inclined arrows indicate the spectrum lines found experimentally. Note the selection rule.*

gas is neon; and no fact is known about this or any other noble gas which would support the idea of rotation. This rotation must therefore be a property of the valency electron, if it cannot be assigned to the core. In the year 1925 Uhlenbeck and Goudsmit put forward the hypothesis of "electron-spin". They assumed that the electron, which must be supposed to have a certain extension in space, is able to rotate about its own axis, just as does the earth every twenty-four

hours. In addition to the principal quantum number n and the azimuthal number k, which is the measure of the velocity with which the electron moves in its elliptical path, we now have a third quantum number which determines the rotation of the electron about its axis. This spin quantum number, which we will call s, can only have two equal and opposite values, according to the direction of rotation of the electron. These quantum numbers must be different from 1, and they are therefore given the values $+\frac{1}{2}$ and $-\frac{1}{2}$.

Each of these motions, the spin of the electron and its orbital motion around the core produces, in common with all moving electric charges, a magnetic field. These two magnetic fields, with their quantum numbers k and s, combine together in a manner analogous to the Zeeman effect, but the combination can only take place according to quantum principles, in amounts differing by whole numbers. This is only the case, when the axis of the electron spin and the axis about which it rotates in its elliptical path are not inclined to one another, as is the axis of the earth to the axis of its path around the sun, but point either in the same or the opposite direction. The quantum numbers then add up in their full amount, and we get in the one case $k+\frac{1}{2}$, and in the other case $k-\frac{1}{2}$. These two total quantum numbers thus differ from one another by exactly unity. The electron energy is slightly different in the two cases, and thus all energy levels must in reality be double.

The levels with the lowest value of k are, however, as experiment shows, single and not double. This empirical fact requires that our formula should lead to only one value of the energy for these levels. This is only possible if the azimuthal quantum number in this case has a value 0. Then we no longer have two quantum numbers which can be added or subtracted, but only the electron spin, and as long as there is no other rotational motion to combine with it, the direction of the spin is a matter of indifference. We can therefore no longer measure the azimuthal quantum number by the values $k=1$, $k=2$, $k=3$ ... $k=n$, but must introduce a new notation, using the letter l, beginning with $l=0$ and continuing with $l=1$, $l=2$ ... up to $l=n-1$. We can then provide a satisfactory

explanation of the doubling of the lines of the alkali metals.

We have therefore to assume for the azimuthal quantum number of the eccentric ellipse the value 0 instead of the value 1 hitherto used. But this is rather serious as regards our model, for this quantum number was also a measure of the length of the minor axis of the ellipse. The choice of the value 0 would mean that the eccentric ellipse becomes a straight line, and that the electron moves to and fro along this axis clean through the nucleus. Such an idea is quite impossible. We have thus won a Pyrrhic victory. We have explained qualitatively the spectra of the alkali metals, including their fine structure, but the valency electron model, a logical development of the Bohr model of the hydrogen atom, is found to possess an impossible characteristic. It evidently fails us in regard to the multiple structure of spectral lines.

If we now consider the elements in the other columns of the periodic table, the multiplicity of energy levels become far greater. An element now has not one only, but several systems of energy levels, many of which are not merely single or double, but multiplex. Our simple hydrogen-like model no longer suffices to explain these more complicated spectra. We have several valency electrons in the outermost, incompletely occupied shell, and all of them take part in the emission of light. Our difficulties are not removed, but made greater.

The value of the Bohr model was nevertheless decisively proved by its success in calculating the energy levels of hydrogen. The principles derived from the study of hydrogen were applied to the element following hydrogen, helium. Various assumptions were made concerning the paths of its two electrons, but no method of calculation gave results which agreed with experiment. When this fact was admitted, the Bohr model could be judged, and the verdict was inevitable that the limits of its validity had been reached.

It became increasingly evident that this model could not be regarded as a reality. It is useful as a tool, even extraordinarily useful. In spite of its contradictions from the point

of view of our physical conceptions, it gives correct numerical results for the quantum numbers, not only of hydrogen, but of all other atoms. Obviously truth of some kind must lie at the back of it, but it clearly fails to represent the actual processes taking place in the interior of the atom. It is painfully necessary to recognise this fact, and to give up the model. But when we bow to this necessity, we are at least freed from the objections attending the impossibilities of the model, both in its earlier and later form: the incomprehensible absence of radiation from the planetary electrons, the electron switch from path to path, the straight line motion of the electron through the nucleus. We shall henceforth only use the model as a useful aid to memory in finding the whole numbers which lie at the basis of atomic structure. It is pointless to attempt, as was at first done, to form exact ideas concerning the position of the orbits relatively to one another, or to discuss the question whether they are to be imagined as all in one plane, or distributed in three dimensions. They cannot be imagined at all. They are a fundamentally wrong picture: but we may perhaps derive even from these inadequate pictures many truths concerning atoms. We will take this as our next task. Beyond it will come one, which we once seemed to have all but accomplished, but which now seems far from a solution, the question of the true structure of atoms. Our previous ideas are insufficient to enable us to attempt it. We shall need a new and quite different idea, which can only be produced by the creative imagination of the scientist.

Let us not be too impatient, but see whether the wisdom already acquired, though it appear somewhat shallow to us, may not still have something of value to yield us.

In considering classification, and attempting to gain some understanding of the spectra of more complicated atoms than those of the alkali metals, we will therefore attempt to make some further progress with our formal atomic model. We will continue to speak of electron orbits without criticising them too closely; they are now to be simply states of the electron, characterised by quantum numbers. We collect such orbits together to form "shells", by which we mean no more than

a more concrete term for states possessing a certain quantum number n. Each shell continues to possess levels, which are characterised by the number l, which we call the azimuthal quantum number. The number l can have all numbers from 0, 1, 2 ... up to $(n-1)$. The levels will be denoted by n_l, as shown in Fig. 39, which now takes the place of Fig. 37 (p. 108).

In the periodic system the valency of the elements increases from element to element by unity. The number of the loosely bound valency electrons must therefore likewise increase each time by unity. The periodic recurrence of the same valency leads us to the conclusion that each level and shell can only take up a certain number of electrons. Each additional electron chooses the level which offers it the strongest attraction, and hence corresponds to the lowest energy value.

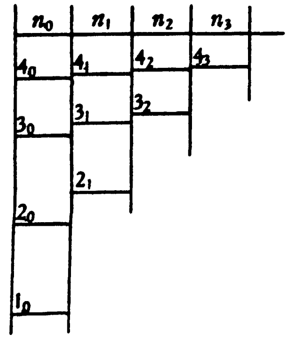


Fig. 39. The energy levels of an element, characterised by the principal quantum number n, *and the azimuthal quantum number* $l = 0, 1, 2, \dots n-1$.

If we expose the atom to light, we may suppose that one or perhaps several electrons are first of all lifted from the loosely bound outermost levels to unoccupied levels lying farther out. The atom is then in an "excited" condition. From this condition the electrons raised to the higher levels can fall back into their equilibrium positions, by emitting as a spectral line the energy they have taken up. Ultimately, when more powerful doses of energy are applied, that is to say, when the atom is bombarded with larger light quanta, an electron is torn completely out of the atom, which is then "ionised". The positive charge of the core is now no longer fully compensated by an equal and opposite valency electron charge, and the atom behaves as if positively electrified. By dosing with very large amounts of energy—for example, by bombarding with cathode rays—it is possible to force an electron out of the inmost and most strongly bound shell, the *K* shell, and bring it to the outermost limits of the atoms. An electron from one of the other fully occupied *L*, *M* ... shells then falls

into the gap thus formed. The result is the emission of a series of spectral lines corresponding to large energy differences $h\nu$, that is to say, high frequency photons. These are the X-rays of the *K* series; they increase in hardness from element to element, corresponding to the growing firmness with which the shell is bound. Thus the X-ray spectra can show no sign of periodicity, which only appears in the case of the electrons situated in the outer, loosely held shells. This fact is already known to us as Moseley's discovery (Figs. 33, 34, pp. 102, 103). From lithium onwards, that is to say, from the element that is the first to possess a fully occupied one-quantum shell, we have a *K* series, and from sodium onwards, we have a softer *L* series. This corresponds to the falling of electrons back into a gap made in the *L* shell. In the case of elements of higher atomic number, we have further and still softer series of X-rays which correspond to the falling of electrons back into gaps in the third, fourth and so on, shells.

In order to explain the multiplex structure of spectral lines, we require here also, as in the case of the alkali metals, a third quantum number s, corresponding to the spin, which can only have the values $+\frac{1}{2}$ and $-\frac{1}{2}$. We continue to imagine the electrons as rotating about their own axis, and indicate the direction of rotation by the sign of s.

The electrons spinning about their own axes, and revolving in their orbits about the nucleus, will produce magnetic fields. All these magnetic fields of the now numerous electrons will mutually influence one another. It turns out that fairly obvious assumptions concerning the nature of this mutual influence, together with the ideas taken over from the simple valency electron model, enable us to comprehend even the enormously manifold and complicated spectra emitted by the higher atoms. We shall not be surprised that the complicated phenomena of the splitting of lines in the magnetic field, which we found incomprehensible in the case of our simple model, require the addition of further rules derived from experiment. To deal with these would be too technical a matter. Only one need be taken note of. The three quantum numbers n, l, s are already insufficient in the case of alkali metals to enable us to understand the splitting up of spectral

lines in the magnetic field. For they do not of course settle the position of the electron in space. It is necessary to add the magnetic quantum number m, which can take on all $2l+1$ whole number values between $-l$ through 0 up to $+l$. Each number distinguishes a particular position in space of the electron orbit, this position being forced upon the electron by the external magnetic field; we thus obtain an explanation of the magnetic splitting of the lines. The electrons of the higher atoms must also be given such a magnetic quantum number.

If the magnetic field is very strong, the resultant splitting up of the energy levels is so great that the mutual influence of the weak magnetic fields produced by the electron motion can be left out of account. Each electron in the atom can then be regarded as independent of all the others, and as being completely determined by four quantum numbers alone, namely:

The *principal* quantum number $n=1, 2, 3 \ldots$ which gives the shell in which the electron is situated.

The *azimuthal* quantum number $l=0, 1, 2 \ldots (n-1)$, which characterises the level in the shell, and originally measures the velocity of travel in the elliptical orbit.

The *spin quantum* number $s=+\frac{1}{2}$ or $-\frac{1}{2}$, which gives the direction of rotation of the electron about its axis.

The *magnetic* quantum number m, which can have all $2l+1$ whole number values, from -1 to $+1$, and determines the position in space of the path into which the electron is forced by the magnetic field.

Let us put together a summary of all possible combinations of quantum numbers for the single atoms:

When $l=0$, $m=0$, and $s=+\frac{1}{2}$ or $-\frac{1}{2}$. This gives two possibilities.

When $l=1$, m is $+1$, or 0, or -1, and in each case, we have in addition $s=+\frac{1}{2}$ or $-\frac{1}{2}$. This gives six cases.

When $l=2$, m may have one of the values $+2$, $+1$, 0, -1, -2, in each case combined with $s=+\frac{1}{2}$ or $-\frac{1}{2}$; altogether ten cases.

When $l=3$, $m=+3$, $+2$, $+1$, 0, -1, -2, -3, together with $s=+\frac{1}{2}$ or $-\frac{1}{2}$; altogether fourteen cases.

We thus have the following different possibilities:

$$\left.\begin{array}{lll} n=1 & l=0 & 2 \\ n=2 & l=0 \text{ or } 1 & 2+6=8 \\ n=3 & l=0 \text{ or } 1 \text{ or } 2 & 2+6+10=18 \\ n=4 & l=0 \text{ or } 1 \text{ or } 2 \text{ or } 3 & 2+6+10+14=32 \end{array}\right\} \text{combinations.}$$

The numbers 2, 8, 18, 32, which state in how many different ways the quantum numbers of the electrons can be combined, are already known to us. They are the numbers of elements contained in each of the successive periods of the periodic system. We can now understand why just these numbers, and no others, govern the development of the chemical elements, if we regard a principle stated by Pauli in 1925 as generally valid. This "exclusion" principle states that:

In any atom, each state determined by quantum numbers can only be acquired by a single electron. This in other words means that no two electrons in an atom may be equal in all four quantum numbers. According to this the total number of electrons in any level must agree with the number of possible combinations.

The number of chemical elements is thus referred to the number of possibilities of stable arrangements of electrons around a nucleus. For all elements, and also for the different states of a single element, the number of possibilities is given by the combination of a few whole numbers, the four quantum numbers and the atomic number of the element. We cannot give any account of the mechanism which prevents the ninety-two electrons of uranium from any of them agreeing in all four quantum numbers. We have no knowledge of the cause of Pauli's principle, for it cannot be deduced from the quantum theory. One further remark must be made. In considering the various possibilities open to the electrons, we have assumed the existence of a strong external magnetic field. But our conclusion also holds in absence of such a field, although we cannot here give the reasons for this fact.

By using an atomic model known to be inadequate, we have nevertheless succeeded in gaining a deeper insight into the structure of the system of chemical elements than we before possessed. We can now proceed to work out the electron

groupings of the various elements, and thus build up those atomic worlds which form the units out of which all those material structures which we can perceive with our senses, are built. From the time of the pre-Socratic Greek philosophers, and Plato, through all metaphysical systems up to Fichte, and after him, attempts had been made to imagine the whole world as built up from one or two elementary unit bricks, as it were; from Being and Becoming, from Body and Spirit, from the Ego and Non-Ego. The bricks which our physics uses, of course only to construct matter, are the proton and the electron. These are themselves mysterious entities, and their mystery can perhaps never be completely solved from the point of view of matter alone. But we do not need to allow this to discourage us, for we may proceed to build up the world as scientists, trusting to our theoretical bases, the quantum numbers of our provisional model, and guided by experimental data, particularly the facts of spectroscopy and chemistry.

For this purpose, we make use of an imaginary experiment suggested by Bohr. We consider the series of elementary nuclei with increasing charges, firstly the hydrogen nucleus with charge 1, then the helium nucleus, and so on. With each new nucleus, we imagine a new electron added, and consider which of the possible unoccupied energy levels offers the new electron the firmest attachment to the nucleus. We then allow it to rotate in the corresponding orbit. The first two elements, hydrogen and helium, will bind their two electrons in the one-quantum K shell, and this is then fully occupied according to the table (p. 116) which we constructed with the aid of the Pauli principle. For this reason, the latter of the two elements helium, is a noble gas. As we proceed further, this construction of the K shell is not changed, but appears in identical form in all following elements.

Our atomic model, if it is to be of any use, must be able to give us correct information concerning the spectra of all elements on the basis of the quantum numbers assumed. We will carry out the argument for the element helium. In its most stable state, both electrons will be found in the one-quantum shell. From the fact that $n=1$ it follows that $l=0$,

and $m=0$. Both electrons are found in the 1_0 level. Since the quantum numbers m, l, and n are the same for both, the spin numbers s must be different according to the Pauli principle. The fundamental state 1_0 is thus only possible for such helium atoms as have electrons spinning in opposite senses about their axes. Helium of this kind is called parhelium. Orthohelium on the other hand, consists of atoms the electrons of which have like spins. These cannot then be alike as regards all other quantum numbers as well; one of them must have therefore $n=2$. Thus for orthohelium a two-quantum state is the most stable and a single-quantum state is impossible.

We can draw a further conclusion. Both electrons behave like minute bodies which rotate about the nucleus, and also about their own axes. Both motions generate magnetic fields, the magnitude of which is measured by quantum numbers l and s. These 2×2 magnetic fields exert mutual influences, and it is only logical to assume that they combine together in such a way that their total effect is again measured by whole quantum numbers. The simplest assumption is that the two spins combine. Parhelium has a total spin impulse $s=+\frac{1}{2}-\frac{1}{2}=0$, while for orthohelium $s=+\frac{1}{2}+\frac{1}{2}=1$. The azimuthal impulses l also add up to give a total impulse $L=0, 1, 2$ .... Finally these magnetic fields measured by the quantum numbers S and L combine together, and form the total impulse of the atom. Its quantum number is one of the whole-number combinations of the numbers L and S. In the case of parhelium, S offers no contribution, since it is equal to 0, and the total impulse is likewise measured by the number L, and hence has one, and only one, value for each value of L. The energy levels of parhelium are therefore all simple, and we hence have singlet terms. The case is different with orthohelium, where $S=1$. Of course, when $L=0$, the total impulse quantum number $=1+0=1$, and the energy states are again singlets. But if L differs from 0, each level is divided into as many levels as there are different whole quantum numbers made up by combining L and $S=1$. The maximum possible number is $L+1$, and the lowest $L-1$, between which we have L; that is to say there are always three possible combinations. The energy levels of orthohelium are therefore,

apart from the case $L=0$, all triplets. Fig. 40 shows the composition of the quantum numbers $S=1$ with $L=1$, and $L=2$. Hence, although our model is not able to give us correct quantitative results, it is nevertheless capable of stating qualitatively and correctly the nature of the energy levels of helium. We find by experiment precisely those spectral lines which are to be expected from these two systems of energy terms. The energy terms of the triplet system of orthohelium prove to be in every case somewhat lower than the corresponding levels of the singlet system of parhelium. Spectral lines which correspond to transitions between the two systems are very rare.

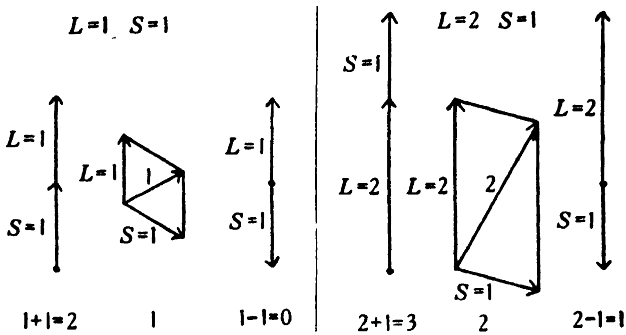


Fig. 40. Combination of quantum numbers L *and* S. *The arrows indicate the quantum numbers by their direction and length.*

In a similar, but much more complicated way, the spectra of the remaining atoms tell us whether the electron arrangement has been correctly chosen.

The electrons of the next two elements (Fig. 41), no. 3 lithium, no. 4 beryllium, must be located in the 2_0 level of the L shell, since the K shell is closed. The two-quantum paths are farther from the nucleus and reach out farther from it. In the case of hydrogen, we know that the major axis of the two-quantum ellipse is four times as long as that of the one-quantum orbit. The new electrons are thus much more loosely bound; this is seen spectroscopically in the fact that lithium spectrum clearly resembles that of a metal with a single outer electron, an alkali metal, as we have already learned. This fact also appears in the chemical behaviour of lithium,

inasmuch as it is monovalent, since the single outer electron is easily detached from it. In a similar way, beryllium is

	1_0	2_0	2_1	3_0	3_1	3_2	4_0	4_1
1 H	1							
2 He	2							
3 Li	2	1	—					
·	·	·	·					
·	·	·	·					
·	·	·	·					
10 Ne	2	2	6					
11 Na	10 Electrons (Neon)			1	—			
·				·	·			
·				·	·			
·				·	·			
18 Ar				2	6			
19 K	18 Electrons (Argon)					—	1	—
20 Ca						—	2	—
21 Sc						1	2	—
22 Ti						2	2	—
23 V						3	2	—
24 Cr						5	1	—
·						·	·	·
·						·	·	·
·						·	·	·
28 Ni						8	2	—
29 Cu						10	1	—
30 Zn	28 Electrons						2	—
31 Ga							2	1
32 Ge							2	2
33 As							2	3
34 Se							2	4
35 Br							2	5
36 Kr							2	6

		4_2	4_3	5_0	5_1	5_2	5_3	6_0	6_1	7_0
37 Rb	36 Electrons (Krypton)	—	—	1	—	—	—			
38 Sr		—	—	2	—	—	—			
39 Y		1	—	2	—	—	—			
40 Zr		2	—	2	—	—	—			
·		·	·	·	·	·	·			
·		·	·	·	·	·	·			
·		·	·	·	·	·	·			
46 Pd		10	—	—	—	—	—			
47 Ag	46 Electrons (Palladium)		—	1	—	—	—			
48 Cd			—	2	—	—	—			
·			·	·	·	·	·			
·			·	·	·	·	·			
·			·	·	·	·	·			
54 X			—	2	6	—	—			
55 Cs			—	2	6	—	—	1	—	
56 Ba			—	2	6	—	—	2	—	
57 La			—	2	6	1	—	2	—	
58 Ce			1	2	6	1	—	2	—	
·			·	·	·	·	·	·	·	
·			·	·	·	·	·	·	·	
·			·	·	·	·	·	·	·	
71 Lu			14	2	6	1	—	2	—	
72 Hf	68 Electrons					2	—	2	—	
·						·	·	·	·	
·						·	·	·	·	
·						·	·	·	·	
79 Au						10	—	1	—	
80 Hg						10	—	2	—	
·						·	·	·	·	
·						·	·	·	·	
·						·	·	·	·	
86 Rn						10	—	2	6	
87 ?						10	—	2	6	1
88 Ra						10	—	2	6	2
·						·	·	·	·	·
·						·	·	·	·	·
·						·	·	·	·	·
·						·	·	·	·	·

Fig. 41. Occupation of levels by electrons.

divalent, and its spectrum is the more complicated one of an alkaline earth metal. Thus the noble gas with its completed shell is always followed by an alkali and an alkaline earth metal, inasmuch as the last electron or electrons added

are held in a loosely bound eccentric ellipse belonging to the next higher shell.

With beryllium, the 2_0 level is fully occupied. The six next elements will bind the added electrons in the 2_1 level, so when we arrive at element no. 10 neon, the whole L shell is fully occupied, according to Pauli's principle. This element is thus another noble gas. In exactly the same way, the 3_0 and the 3_1 levels fill up with two and six electrons from the alkali metal sodium (11) to the noble gas argon (18) and these new elements are quite similar chemically and spectroscopically to their eight predecessors (Fig. 41).

Now we come to an interesting point. With argon, the three-quantum M shell is not yet quite filled up. The whole 3_2 level remains unoccupied. The next elements, potassium and calcium, are true alkaline and alkaline earth metals in their whole chemical and spectroscopical behaviour, and hence the two next electrons will not be bound in the 3_2, but in the 4_0 level. Why is this level the more favourable from the energy point of view? Here again we find that our easily understood atomic model, in spite of the internal contradictions which we know to exist, must contain a large percentage of truth. For here again it gives us a plausible answer. As the charge on the nucleus increases, all orbits already present are attracted closer to the nucleus, and the ellipses which stretch far out and at the same time penetrate into the core, are particularly influenced; hence an electron is bound more firmly in them than in the circles and nearly circular ellipses, which are somewhat protected by the core electrons lying inside from the attractive action of the nuclear charge.

The 3_2 stage is only filled after the 4_2 has been filled. Scandium, no. 21, first binds a 3_2 electron. This continues in the case of the following elements, where the 4_0 and the 3_2 levels compete with varying success for the last electrons. On account of the electrons thus inserted, these elements differ considerably, both chemically and spectroscopically, from those standing above them in the columns of the periodic table. With copper (29) and zinc (30) the structure is completed. These two elements bind one and two electrons

respectively in the 4_0 level like potassium and calcium, to which they are also related. The next six elements then fill up the 4_1 level. With no. 36 krypton, we again arrive at a noble gas.

The same process repeats itself in the eighteen elements of the following long period. In rubidium and strontium the 5_0 level gains its two electrons. Then we have a further ten elements in which the 4_2 level fills up. Silver (47) and cadmium (48) are elements allied to copper and zinc; in the case of the six following elements, the electrons are taken into the 5_1 level. With no. 54 we reach again a noble gas, xenon.

We now come to the interesting long period with thirty-two elements. It begins just like the others with an alkali metal, caesium (55), and the alkaline earth barium (56) in which the 6_0 level is occupied by two electrons. With lanthanum (57) the first electron enters the 5_2 level, which is still quite empty, and we reach a truly remarkable feature of the table. We find that the next fourteen elements, the rare earths, are very closely similar to one another in chemical behaviour. This can only mean that the arrangement of their external electrons must also be very similar, since this decides all the chemical and physical properties commonly met with. But in their case, the X-ray lines advance, as with all other elements, according to Moseley's law (Figs. 33, 34). Hence they form no exception to the rule that a new electron is added with each new element. Here, however, this addition can only take place in the interior, since the outer set of electrons remains alike from element to element; hence it must be the 4_3 level, which is still entirely empty, that is being filled up. Its complement of electrons is fourteen, according to Pauli's principle. Hence fourteen rare earths must exist. Commencing with element 72, the 5_2 level fills up step by step, finishing with no. 79 gold, and no. 80 mercury. The level 6_1 can now be filled up, element by element, from 81 to 86. Element no. 72 had not been discovered in the year 1922, when Bohr put forward views similar to the above, at a time when the Pauli principle was not known. It had been supposed until then to be a missing rare earth. Bohr suspected that this could not be true, since no fifteenth place

existed for such an element; he predicted that it would be similar to zirconium (40), since it has a similar arrangement of external electrons. Whereupon Hevesy and Coster shortly afterwards discovered the new element in zirconium minerals, and gave it the name of hafnium. This prediction of its properties is one of the finest successes of the Bohr theory. With no. 86, we come to a noble gas radon, or radium emanation. In the following six elements, the 7_0 and 6_2 levels are occupied. We do not know why no elements of higher atomic number than 92 exist.

This formal atomic model, which allows us to give to the quantum numbers derived from Bohr's theory a kind of concrete reality, leads us to a qualitative understanding of the way in which the chemical elements are built up. We are reminded of the saying of a great mathematician: Whole numbers were made by God, while all the rest is the work of man. The first experimental physicists known to history, the Pythagoreans of Sicily, made a very great discovery when they showed that the musical scale is determined by whole numbers. Θεὸς ἀριθμετίζει, God counts.

We can now proceed to make a further application of Bohr's theory, and attempt to explain the nature of chemical attraction. What kind of force is it which holds together the atoms in a chemical compound? The answer to this question would be fundamental of the whole of chemistry. Bohr's quantum theory had given physicists a great deal of fresh information concerning what happens inside the atoms, and one would therefore expect it to have something decisive to say concerning this fundamental question of chemistry.

Up to Bohr's time, a very great deal had been found out about the manner in which chemical forces act. The composition and mutual interaction of very complicated compounds, such for example as the dyestuffs of organic chemistry, was known very exactly, but nothing was known about the nature of the forces between the atoms. Expressions such as strong or weak "affinity" of elements, notions such as "saturated" and "unsaturated" compounds, were useful anthropomorphic pictures, but nothing more.

Davy and Berzelius had about 100 years ago put forward

the view that the forces acting between atoms are electrical in nature, acting between atoms possessing an opposite electric charge. But this theory could not be upheld. More especially, it was quite incompetent to deal with organic chemistry, then in rapid and extensive development. For in this case, we are dealing almost entirely with combinations between like atoms, particularly carbon, which unite to form chains and rings. It is impossible to assume that carbon atoms sometimes have a positive and sometimes a negative charge.

A few years after Bohr's first investigation, Kossel in 1916 reverted to Berzelius's ideas in a revised form, using the Bohr atomic model. Let us look once more at Fig. 35, p. 105. As we pass along the series of elements from one noble gas to the next, one electron after another is built into the structure of the atom. These are the elements with levels not completely filled with electrons. Kossel assumes that the electron configurations characteristic of the noble gases are easily formed in the case of other elements as well, since they are of the stablest. The elements next following a noble gas easily part with the excess electrons of the outermost and only partly occupied levels; sodium easily loses its solitary outer electron, magnesium not quite so easily its two outer electrons, while aluminium can part with three, and silicon with four. This process results in the formation of "ions", that is to say atoms with a positive excess charge, due to the positive nucleus, of 1, 2, 3 or 4 electron units.

The following elements will also be able to give up five and more electrons, and thus behave like metals in a similar manner to their predecessors, but they can more easily take on the configuration of a noble gas, namely that next following, by capturing from somewhere or other the electrons required to fill up their outer levels. This is the characteristic of a non-metallic element. Silicon needs to catch four electrons, phosphorus three, sulphur two, and chlorine, as well as the other elements just preceding a noble gas, only one. If therefore sodium atoms and chlorine atoms meet, the sodium atom will readily part with its external electron, which will then fit into the only free space in the external

shell of chlorine, and both elements thus acquire complete outer shells, and hence the configuration of a noble gas. At the same time, they are no longer electrically neutral. Chlorine has an excess negative charge, sodium a positive charge. These charged ions attract one another according to the ordinary laws of electricity, and form the compound sodium chloride, common salt. This conception of the structure of sodium chloride can be proved by dissolving it in water, when we actually find that electrically charged sodium and chlorine atoms are present, as chemists had long ago discovered. We may now add that the electrical attractive forces between oppositely charged bodies, such as our ions, are known to be sometimes eighty times smaller between bodies in water than in air, and hence the sodium and chlorine ions separate from one another when common salt is dissolved in water, and can then be separately detected.

In a similar way, Kossel's theory can be tested in the case of other compounds. The theory ascribes to all elements the power of dropping electrons from, or adding them to, their atoms, and thereby acquiring a more stable, and in particular a noble gas, configuration. Sulphur, for example, is able either to lose six electrons, or to take up two, and in both cases, ions are formed possessing a closed noble gas shell. The first case will be realised when a sulphur atom meets other atoms capable of adopting the six electrons, for example three oxygen atoms; the second case occurs when it meets atoms able to supply two electrons, for example two hydrogen atoms. We thus get in the first case the well-known compound sulphur trioxide SO_3, and in the second sulphuretted hydrogen, H_2S.

This route leads to an understanding of many chemical facts. We will not follow it further, for to do so would require considerable knowledge of chemistry. A more serious test of Kossel's theory would call for numerical calculation of the forces of chemical attraction. Such calculation has frequently been attempted. The volatility and conductivity of compounds have also been calculated, and the relations between different properties, such as between solubility and certain electrical properties characteristic of a substance. Such cal-

culations were not always successful, but they frequently gave good results. They were ultimately based upon the Bohr atomic model, and we are already accustomed to the fact that it fails in the end for quantitative purposes. Also, chemical compounds exist which we are not able to understand on the basis of Kossel's theory. They are those which presented insuperable difficulties to Berzelius, namely, compounds formed of like atoms, as found in the whole of organic chemistry. Even so simple a compound as the hydrogen molecule, which consists of two hydrogen atoms, could not be successfully constructed out of two protons and two electrons, and made to give the correct numerical values for hydrogen gas. The theory failed entirely in these cases of simple combination between like atoms.

We have now come to an end of our survey of the capabilities of Bohr's atomic theory, which in essence consists of the two peculiar postulates with which Bohr limited the validity of classical physics as applied to the paths and radiation of atomic electrons. This "classical" quantum theory, constructed from a mixture of classical and quantum ideas, nevertheless had a triumphant career. What a change it brought about in our ideas! When Bohr put his atomic model forward in the year 1913, he could only combat the incredulity of other physicists by proving that he could exactly explain the spectrum of hydrogen. Then followed all the great successes. Wherever the theory was applied, it opened, as though with a magic key, doors that had hitherto resisted every attempt to unlock them.

It had thus taken a little less than a decade to discover that all the complex spectra of the elements could be explained qualitatively at least, by a theory which shows how they are to be based upon four whole numbers. Furthermore, some insight had been gained into the chemical nature of the elements, and into the periodic system discovered so long before. This new knowledge was strikingly attested by the successful prediction, à la Mendeléeff, of the properties of an undiscovered element, hafnium.

The real basis of all this new development was Planck's constant of action h. This determined the position of the non-

radiating electron orbits, or in later and more abstract terms, of energy levels, and it also settled the frequencies of the spectral lines. Bohr's theory thus proved itself in no way inferior, in elegance and power, to the historical triumphs of classical physics. In the heyday of the latter, at about the end of the preceding century, science had offered an explanation of the physical world in terms of a dualism, matter and radiation. Matter was made up of minute electrified corpuscles while radiation was wavelike in nature, though not wave motion in a medium, but alternating electric and magnetic fields in space. After Bohr's time, our picture of the world is no longer so neatly rounded off. It is altogether more mysterious and less concrete. But we know a very great deal more.

In the case of light, there was now every reason for hesitating between a wave and a corpuscular theory. Optical phenomena can be explained, for the most part, only by a wave theory, but another part, including all phenomena of light emission and absorption, can only be explained by assuming that light consists of corpuscular bodies, the photons. Matter was still thought of as built up of minute bodies, protons and electrons. The investigation of cathode and X-rays, and of radioactive substances, had so plainly exhibited the action of single atoms and atomic particles, that no one could doubt the existence of electrons and protons. Electrons behave, say in an X-ray tube, exactly like minute solid electrified particles, and even when they rotate around atoms in orbits more distant from the nucleus, they still radiate approximately in the same way as classical physics predicts for rotating negatively charged particles. But when they are more closely attached to the atom, something seems to go wrong with them; classical ideas fail us, we begin to get into difficulties with our clear mental pictures. These now lead to impossibilities, such as ellipses degenerating into straight lines, which would involve the electrons in collisions with the nucleus.

Our endeavours to gain a clear picture of the motions of electrons bring us to a limit which we are unable to pass, in spite of our liveliest endeavours. We have not succeeded

in constructing an atomic counterpart to the macrocosmic solar system, which is a perfect example of the power of scientific thought carried out on the basis of strict causality. Our atomic planetary system contains as an important element the Planck constant h, which plays no part in macrophysics. Hitherto, no success has attended our efforts to reconcile it with the fundamental ideas of classical physics.

It is a striking fact that the position occupied by an electron in its orbit at any given moment, never enters into the formulae which explain spectral lines. Obviously, it is of no importance in this respect. Here we have a complete contrast to the astronomical case, where, for example, the spring tides are determined by the position occupied by moon and earth in their orbits. Must we assume that the position of an electron on its path has, perhaps, no physical meaning at all? We are quite unable to determine this position. One, or a few, electrons apparently fill the whole orbit, and Pauli's principle tells us how many can be fitted in. They seem to be spread over the whole orbit, as light is spread out by a fine slit. Is it perhaps foolish to ask where an electron is at a given instant, and what is its velocity? Perhaps these questions are as meaningless as that famous topic of discussion by the mediaeval Schoolmen: how many angels can find room to dance on the point of a needle! Without a doubt, the first crude notions we form concerning either angels or electrons will not carry us very far. At first, nothing seems more simple than the motion in its orbit of the smallest particle of matter, an electron; but it is, in fact, completely mysterious.

In point of fact, the original Bohr model fully admitted this. However, in the years of its unexampled success, all doubts about it were laid on one side in the hope that a solution would be found for them; but we have seen that, in point of fact, they finally emerged more clearly than ever. Why do not the electrons radiate in their stationary orbits? And when they radiate, why has the frequency of the radiation nothing to do with the frequency of rotation in the orbit? Furthermore, the "switch" of the electrons is also a very doubtful business. For they must of course jump through forbidden territory; how long are we to suppose them to stay

in it, and at what point of this period do they begin to radiate? Another extremely unsatisfactory point has already been mentioned, namely the mixture of completely irreconcilable classical and quantum principles. We can certainly apply to Bohr's theory the oft-quoted line from *Hamlet*: "Something is rotten in the state of Denmark". This theory is like a thin sheet of ice, which has covered in a night the surface of a lake. If we tread very gently, we may pass over it to the shore of a new truth, yet this truth does not seem to have been reached by legitimate methods. But physical theories are not ends in themselves. It does not really matter whether the pictures which they present to us are true in every detail. The question is whether they advance our knowledge of what actually happens in the world. Even incorrect theories may do this; all theories are false *sub specie aeternitatis*, for they are subject to our human limitations, and yet attempt to divine something of the Infinite. But although the physicist must needs sacrifice his theories upon the altar of truth quite frequently, and with apparent callousness, they are nevertheless not a matter of indifference to him. For the more deeply we penetrate into the phenomena of nature, the more closely is the picture we form adapted to reality, that is to say, the more complete is our theory.

As Sommerfeld remarked at Innsbruck in 1924: "The model of the atom is more a device for calculation than a reality of existence." Rutherford expressed himself in those years of doubt as follows: "It may perhaps be necessary at least to make further changes in classical views." Planck already warned us in 1921, in his Nobel address: "It is true that the introduction of the quantum of action does not lead us to construct a real quantum theory. Indeed, it may be that the road which Science will have to travel to the latter point is not shorter than that which led from the discovery of the velocity of light by Olaf Römer to the foundation of Maxwell's theory of light." All leading physicists were sceptical at that time.

Bohr above all was sufficient of a revolutionary to doubt whether the concepts which we derive from the macroscopic world are of any use in enabling us to form a picture of the

micro-world of the atom's interior. Helmholtz once said, speaking from the standpoint of classical physics: "English physicists such as Lord Kelvin, with his theory of vortex atoms, and Maxwell, with his assumption of a system of cells with rotating contents by which he used to attempt to explain electromagnetic processes mechanically, obviously felt more satisfied by explanations of this kind than by the mere general representation of the facts and their laws, such as is given us by the differential equations of physics. I must admit that I myself have hitherto preferred the latter form of statement, and have felt safest in doing so. But I should not like to raise any objection in principle to the road taken by physicists as eminent as those I have mentioned." Bohr, after more than ten years of failure in all attempts to construct comprehensible pictures of the processes taking place in the atom, felt justified in raising fundamental doubts of this kind. His school became convinced that all pictures derived from macrophysics must be abandoned; at the very least one thing, they thought, was clear, namely that the mixture of classical and quantum ideas, which was Bohr's original basis, must be abandoned.

It was necessary to start building afresh. This conclusion made those years of physical investigation exciting and interesting. We were clearly standing once more face to face with the most fundamental mysteries of matter. How were we to imagine its smallest particles to be constituted? It is certainly not legitimate simply to carry over the known laws of motion of bodies large enough to be perceived by our unaided senses, into our dealings with electrons. A new and more profound science of mechanics must be sought for. This must lead to the laws of quantum physics when applied to the smallest particles, and when we then apply it to larger bodies, the laws of ordinary mechanics must result. For the processes going on inside the atom cannot be described simply by the picturable motion of electrically charged particles in space and time. The subatomic world must be attacked in a different way. The thorough exploration of the road travelled by a generation of physicists, led by Bohr, at the cost of more than ten years of labour, was necessary to

convince us that the semi-classical route, like the classical, might lead us to many an interesting view point in the unknown land of physics and chemistry, but would not bring us to our goal. After this digression, we are more willing to be led along steep and difficult paths. Indeed the necessity for a complete revolution in our ideas would otherwise be scarcely comprehensible; as it is, we are convinced that it is essential to seek an entirely new road.

Until this is found, as von Laue said in 1921: "We are in an extremely difficult position. We have only one consolation....The previous line of thought does not lead to a solution of the contradiction between the quantum and the older points of view. How soon we shall succeed in solving this problem, whether great experimental discoveries are perhaps necessary, or only the genius to find a new idea, no one can say. But since it must be possible sooner or later to resolve the contradiction, we can feel sure that physics at the present time is on the verge of a quite fundamental advance." This advance was made, as a kind of Jubilee celebration of the first foundation of the quantum theory, by the work of Heisenberg and his collaborators, and of de Broglie and Schrödinger. We will now proceed to consider this new departure.

CHAPTER VI

Quantum Mechanics and Matter Waves

It was a most adventurous period, full of surprises and disappointments, of successes, and of profound difficulties, the discussion of which has led us to examine the foundations of all physical knowledge.

W. HEISENBERG.

Bohr's atomic model, as we have seen from the theory of the periodic system and the spectra of the higher atoms, is no more than a useful working hypothesis. It is fitted to give us a provisional and qualitative survey of atomic processes, a survey containing many correct details, but it is incapable of supplying a consistent mechanical picture agreeing with all our experimental experience. It supplies no answer whatever to the question of how we are to imagine the electrons existing in the atom. We have heard Bohr's pessimistic view. Dealing with quantum experiments, he said in a lecture in 1925: "We may conclude from these results that the general problems of the quantum theory cannot be dealt with simply by altering our mechanical and electrodynamic theories in a way which can be described on the basis of ordinary physical concepts; we have to face the fundamental failure of the spatial and temporal pictures, by means of which we have hitherto attempted to describe natural phenomena." For this reason he recommends us to "abandon mechanical pictures in space and time". These appear insufficient to describe what we call quantisation. Perhaps it is simply naïve to say: "The inside of the atom looks like so and so." For perhaps it does not "look" at all. It may be that in this microscopic world, there is nothing resembling what we see with our eyes in the world around us.

These ideas of Bohr's were followed up by the German physicist Heisenberg. We become more closely acquainted with his point of view by considering the fact that our theory has hitherto always attempted to tell us too much. The orbits

of the electrons, their momentary positions, their times of revolution, are all magnitudes that we cannot even deduce indirectly from the data of spectroscopy. This mixture of classical physics and quantum theory, this "classical quantum theory", makes far too many unprovable statements about the processes going on in the atom. For we can only actually observe the frequencies and intensities of spectral lines, together with the value of the individual possible energy levels as given by the method of Franck and Hertz (p. 62). We are told nothing about the orbits and periods of the electrons.

Let us therefore turn aside from this hypothesis and recall the true problem of physics. We must find the means of dealing with mathematical exactness with our spectroscopic observations. This mathematical statement, if correct, must enable us to predict the results of experiments not yet made. A test will then show whether our formula is correct, but to go further and demand that this formula should be based upon a model of the atom comprehensible by analogy with the objects of the everyday world, is perhaps too much to ask. A correct idea is not necessarily concretely imaginable. Perhaps we may have to be satisfied with a mathematical point of view, that is to say, one giving the way in which the world works in purely formal terms. We should then allow Goethe's Philistine to be right: "No mind created on earth penetrates into the workings of nature." That is to say, we can never attain to an understanding which goes beyond simple mathematical statement. "The sole problem of theoretical physics is to make predictions which can be compared with experience" (P. A. M. Dirac).

Heisenberg therefore requires us to construct a theory which deals only with what can be actually observed. The failure of the first quantum theory leads us to the philosophical theory of knowledge known as "positivism". Many years previously, Mach had upheld this point of view in physics. At that time he rejected all detailed features of the atomic theory, since atoms could not be observed. It later appeared that his scepticism went too far. But Heisenberg's criticism of mechanical atoms with electron satellites had from the start more right on its side, for these pictures had

obviously misled us. Hence a critical Cartesian point of view is at least very reasonable to start with. We must begin by abandoning every feature of our atomic model which is not unquestionably correct. But for the physicist, the unquestionably correct is, before all things, that which is ascertained by experiment. The mathematical description of this is the final goal of positivism, and Faust's desire for an insight into the forces and germs of all action is nonsense to the extreme positivist. We shall discuss later the justification of this point of view.

Physically the problem is quite clear. We have to create a new "quantum mechanics". It must be so constituted that Bohr's postulates, so successful in practice, appear as natural laws, while the laws of classical physics, which know nothing of the postulates, must appear as the limiting case for large dimensions. The state of an atom will no longer be described in terms of the unobservable position and impulse of its electrons, but of the measurable frequencies and intensities of its spectral lines.

Every frequency of a spectral line depends upon two energy levels, let us say the *n*th and the *m*th, for it results from a "switch" from one to the other. It is a matter of indifference to know the nature of the actual musicians which play to us the optical music of the atoms; Heisenberg simply imagines that each performer plays only one note with a definite loudness. Each of these musicians is represented by a mathematical expression, symbolised by q_{mn}, which includes the intensity and frequency of the spectral line, just as these same quantities are included in the formulae of the ordinary physics of sound. These musicians are combined together, according to the beginning and end states m and n of the "switch" considered, to form an orchestra. The mathematician calls such an arrangement a "matrix".

$$\left.\begin{matrix} q_{00} & q_{01} & q_{02} & q_{03} \cdots \\ q_{10} & q_{11} & q_{12} & q_{13} \cdots \\ q_{20} & q_{21} & q_{22} & q_{23} \cdots \\ q_{30} & q_{31} & q_{32} & q_{33} \cdots \\ \cdots & \cdots & \cdots & \cdots \ \cdots \\ \cdots & \cdots & \cdots & \cdots \ \cdots \end{matrix}\right\} = q.$$

This whole matrix contains all the numerical statements which can be made about the atom. It takes the place of the numbers which formerly determined the position of an electron. Heisenberg also sets up matrices of this kind to describe the velocity and impulse of the electron. While the energy values of the atoms had previously been calculated, by the application of the laws of mechanics, from the position and impulse, mathematical research had already provided us with the rules for calculating with matrices. These rules differ in part from the rules which hold for figures. In multiplying two matrices, for example, the factors may not simply be interchanged. In the case of numbers, this may always be done. 3×4 is always equal to 4×3, but this is not true of two matrices p and q. $p \times q$ has a different value from $q \times p$. Heisenberg now assumed that this difference, when a product is formed from a position and an impulse matrix, is equal, apart from a numerical factor, to the constant h. This single assumption—some assumption must be made at this point—takes the place of the unexpected and disconcerting two postulates of the earlier quantum physics. If we assume the opposite view to Heisenberg, namely that the products qp and pq are equal, as is true of the numbers with which classical physics operates, we get an approximate result only, since we take the quantum of action to be zero, whereas it has in reality the very small but finite value $6{\cdot}55 \times 10^{-27}$.

Bohr had long ago sought for a "principle of correspondence", as he called it, between our classical pictures of processes, and the actual processes described by quantum physics. Heisenberg's matrix mechanics is the actual carrying out of this programme.

The energy calculations made on the basis described lead, one after the other, to the energy levels already known for hydrogen, and in the case of the spectra of the higher atoms, it was possible to calculate correctly the rules already known, and mostly empirical in origin, without any further additional assumption. Heisenberg and his collaborators, particularly Born and Jordan, succeeded in principle in carrying out a logically consistent formulation of the whole of physics.

His method was nothing new and unaccustomed in physics.

Newton, as we know, created for himself, in the differential and integral calculus, a new mathematical method, when he found that the concepts of pressure and collision current at the time were not adequate. Maxwell did the same when he formulated mathematically the laws of electromagnetism, and Einstein when he stated the relativity theory. The necessity for new mathematical forms repeatedly arises when formulae have to be fitted to newly discovered facts. The calculations of matrix mechanics are very difficult to picture; but this is better than the deceptive and erroneous concreteness of previous theories. The actual working out is also very difficult, and this hinders the application of the theory. The question of the structure of the atom is avoided. It remains a riddle. But proof has been brought that atomic problems can be solved even if we abandon pictures derived from the world of visible objects. It is true that more profound reasons are required to explain why the concepts of macro-physics fail. Heisenberg himself was able two years later to give us these reasons. We will deal with them later. The Heisenberg theory is the exact opposite of what Goethe understood under the name of natural science. Both Heisenberg and Goethe go back to something which Goethe called the primal phenomenon (*Urphänomen*), beyond which it is meaningless to question further. Both abstract from the results of experimental observation by our senses an idea; for Goethe this idea is ultimately imaginable in terms of our sense impressions, and from it all phenomena can be deduced, for Heisenberg it is a mathematical abstraction, from which all phenomena can be calculated.

The doubts of Bohr and Heisenberg concerning the atomic model were not shared by all physicists. Other attempts were also made with the same end in view, a theory combining with complete consistency the results of classical and quantum physics. The most important and most fertile idea for the understanding of atomic processes by classical concepts, is the idea of wave particles or matter waves, which was stated by Louis de Broglie in 1925 and developed in the years following by Schrödinger into his "wave mechanics".

All matter consists of minute corpuscles. They possess

energy and impulse; mechanics states the laws according to which they can exchange these. Matter extends in space and time. Mechanics also gives us the laws of motion of matter with a certainty and accuracy which has made this science a pattern for scientific investigation. But in spite of our admiration for the accuracy with which astronomers calculate the paths of the planets in the sky, we are completely disappointed when we find that mechanics fails to state correctly the motion of the electrons upon the minute paths in the interior of the atom. We have already remarked that no experiment can tell us at what point on its journey round the atom the electron is to be found at any moment. It appears, indeed, as though it might be spread over the whole orbit.

May not this perhaps form a starting point for our solution? We may recollect that the case of light was exactly similar. We assumed that light, like matter, was made up of corpuscles, called photons, which could also exchange energy and impulse according to the laws of mechanics. When we consider the propagation of light in space and with time, we can regard its rays as the tracks of these flying corpuscles. Newton already had this idea. It is true that a wave theory of light also exists. Nevertheless, even to-day, we follow Newton. When we investigate the passage of light through the lenses, prisms, or mirrors of, say a telescope or microscope, and wish to find where it will arrive and what sort of images it will form, we do not think of it, at any rate apart from a very refined investigation, as a wave; we find it much simpler to imagine it as corpuscles flying along the directions of the rays. But when we pass the light through an extremely fine slit, this notion of tiny moving corpuscles breaks down. It is useless to talk of rays of light any longer. The light spreads far out on both sides beyond the slit, over the space lying beyond it. It behaves like waves which interfere with one another.

Thus the ray conception, geometrical optics as it is called, tells us the path of light correctly only as long as we are dealing with openings large enough to allow us to neglect the wave-length. But wave optics is valid both for large and for small dimensions, without limit. It includes geometrical

optics, and gives us, as we have already seen, the same results when dealing with openings large compared with the wave-length. It only gives different results in spaces of the smallest dimensions, and these results are perfectly correct.

How if something of the same kind were true concerning matter? Perhaps mechanics as hitherto known to us meets with such difficulties in the interior of the atom because, like geometrical optics, it treats matter as made up of minute corpuscles. And although this notion is so successful in the macroscopic world, just as it is in geometrical optics, may it not be that the motion of the electron in the minute dimensions of the atom ought rather to be treated like the problem of light in a narrow slit? It may be that in both cases we ought to think of waves rather than corpuscles. This would mean that we should pass from a corpuscular, ray mechanics to a wave mechanics, just as we pass from Newton's corpuscular optics to the wave optics of Huygens.

This brilliant but extremely daring hypothesis, which proposes to explain Bohr's quantum postulates and to solve the riddle of the atom by regarding the electrical matter moving inside the atom as a system of waves, is due in the first place to de Broglie, and following him Schrödinger. Once before, a hundred years earlier, Hamilton had entertained ideas in the same direction, when seeking a connection between optics and mechanics. At that time, he was unable to solve the problem. The matter now stands as follows. In the case of light, the connection between the energy E and impulse p, which characterise the corpuscle, and the wave-length λ and frequency ν, which describe the wave, is given by the equations

$$E=h\nu, \qquad p=\frac{h}{\lambda}.$$

In a corresponding manner, de Broglie now associates with each material particle, possessing energy E and impulse p a wave, the frequency ν and the wave-length λ of which can be deduced from these same equations.

The electron in Bohr's model of the hydrogen atom is not to be imagined as a corpuscle travelling around a circle or ellipse, but as a wave permanently occupying a whole circle,

so that there is no point in enquiring as to its position on the circle, since it is at every point of it all the time. This electron has a definite wave-length, with the result that it can only fit into certain circles. Fig. 42*a* shows an impossible case. A whole number of waves, say n, must just find room around the circle, as shown in Fig. 42*b*. Hence only circles of certain

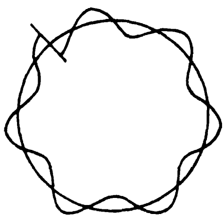


Fig. 42a.

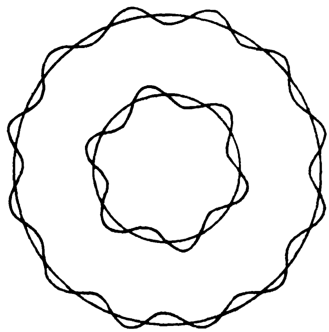


Fig. 42b.

Electrons as waves in the Bohr orbits (according to de Broglie).

definite sizes are possible paths for the electrons, and all those lying in between are excluded. Since the circumference of a circle $= 2\pi r$, we must have

$$2\pi r = n\lambda,$$

where n is any whole number. In the figure $n = 6$ and 12. All paths agreeing with this equation are possible in the case of a wave electron. But since

$$\lambda = \frac{h}{p}, \quad 2\pi r = n\frac{h}{p},$$

and hence
$$p = n\frac{h}{2\pi r}.$$

In this formula, the condition for possible paths is so expressed that the λ characteristic of the wave nature of the electron is replaced by p, the impulse characterising our original corpuscle electron. And the astonishing thing is that we obtain precisely that condition for "permitted" orbits already stated by Bohr (Chap. IV). Bohr was able to give no reason for it,

but it led to correct results. De Broglie now shows that it follows of necessity, when the electron is imagined not as a charged mass point, but as a wave filling the whole orbit. The postulate of stationary orbits is now comprehensible on classical lines. Waves which are to remain permanently in closed paths cannot be otherwise than quantised. This was a notable discovery. The constant h continued to maintain its fundamental position as regards the workings of the atom. It now appears as the number which links the magnitudes characteristic of the corpuscle point of view, p and E, with those characteristic of the wave theory, λ and ν.

If the position occupied by matter in motion cannot be correctly calculated by assuming that it is a body, but only by assuming it to be a wave, why have we never noticed this before? Why in dealing with the motion of electrons in a cathode ray tube, or the motion of larger particles such as whole atoms or molecules, have we never got into difficulties through regarding them as bodies, and taking no account of their hypothetical wave nature, which no one had imagined or suspected before de Broglie? The only possible answer is obtained by considering the analogous case of light, the wave nature of which can be ignored, excepting when we are dealing with a narrow slit or very fine grating. In all other cases light can be treated as a shower of particles; for its wave-length is so small as to be negligible when we are determining motion and position.

What is the wave-length of an electron? Since $p=mv$, and $p=\frac{h}{\lambda}$, $mv=\frac{h}{\lambda}$, and

$$\lambda=\frac{h}{mv}.$$

Now since $h=6{\cdot}55\times10^{-27}$, a body of mass $m=1$ gram, moving with velocity $v=1$ cm. per second, would have a wave-length $\lambda=\frac{h}{1\times1}=6{\cdot}55\times10^{-27}$ cm., which is more than a billion times smaller than the wave-length of the hardest gamma rays.

It is clear that in this case no experimental test of the wave nature is possible, for the experiment requires the use of a grating the spacing of which is of the order of the wave-

length. But no mass of 1 gram known to us is small enough to pass through a suitable grating, even if the latter could be made. But the case is different with minute masses such as atoms or electrons, which are readily obtainable travelling at low speeds and therefore possessing a long wave-length. As soon as this wave-length approaches that of X-rays, not to say light, it should be possible to observe the wave nature of this moving matter, for if it be sent through a fine grating, it should not continue to travel forward in a straight line, as would corpuscles, but rather should be bent aside as are light waves. Furthermore, material particles of this kind should be capable of extinguishing one another when they meet aright, that is to say, when a wave crest of one coincides with a wave trough of another. But it is certain that nothing of this kind had been known to occur.

Let us consider how an experiment with cathode rays would be likely to turn out. They have, as we learned on p. 22, a mass $m=9\times 10^{-28}$ grams and a velocity which depends upon the voltage applied to the cathode tube and varies with the ordinary voltages used between 10^7 and 10^{10} cm./sec. We at once calculate that for $v=10^8$ the wave-length should be

$$\lambda=\frac{h}{mv}=\frac{6\cdot 55\times 10^{-27}}{9\times 10^{-28}\times 10^8}=7\times 10^{-8}\text{ cm.},$$

that is to say, of the same order as that of X-rays. It should therefore be possible to diffract these electrons by crystals just as X-rays are diffracted. The first experiment was made by Davisson and Germer in 1927, and was quickly followed by others. Fig. 43, opposite p. 142, shows a photograph taken by the Japanese physicist Kikuchi in 1928.

He obtained this by sending electrons through a plate of mica only 10^{-5} cm. thick. The atoms of the mica act like a flat grating, such as, for example, a fine fabric through which one looks at a burning candle. The pattern seen on the photograph is not, of course, formed simply by the electrons flying through the holes between the atoms of the mica, for they are much closer together. What we have is a diffraction pattern resulting from the interference of electron waves, and

comparable with that given by light waves in Fig. 19, p. 40. The pattern of light and dark markings is exactly that to be expected according to de Broglie's ideas. If a plate ten times this thickness is used the crystal acts like a three-dimensional grating. We should then expect results similar to von Laue's diffraction photographs (Fig. 21, p. 50), namely single points intensified by interference, and determined by the intersection of ellipses, circles, parabolas, and hyperbolas, and this is in fact what we get. We see single points lying on circles or nearly circular ellipses, precisely similar to those to be expected from the structure of mica as already determined by X-rays. The light and dark lines on the picture can also be explained, but this would carry us too far. If we use finely powdered crystal, we obtain photographs similar to Fig. 23 (p. 51) obtained by X-rays. Fig. 45 shows two photographs obtained by Thomson by means of electrons sent through thin foils. Conversely, electrons may be used to determine the structure of crystals, and the agreement with the results obtained from X-rays is perfectly satisfactory. Cathode rays are now used industrially in the same way as X-rays for the examination of materials. On account of their intensity, it is possible to use much shorter exposures than are required for X-rays. Stern and Estermann showed in 1930 that not only electrons but whole molecules, such as hydrogen and helium, can be diffracted.

This astonishing idea of de Broglie's has thus been proved true. Small particles of matter in motion behave like waves. For just as we regarded the diffraction of light as a proof of its wave nature, so must we now regard these experiments as proof of the wave nature of matter.

This discovery that electrons are just as much waves as are photons, leads us to expect great things of an electron microscope. In the ordinary microscope, light rays are bent by lenses in such a way as to produce an enlarged image of an object. In the electron microscope, we use instead of light waves electron waves, which may be projected from an iron cathode by a pressure of say 70,000 volts into a suitable evacuated tube. In place of glass lenses, the electron rays are deflected either by electrically charged discs perforated in

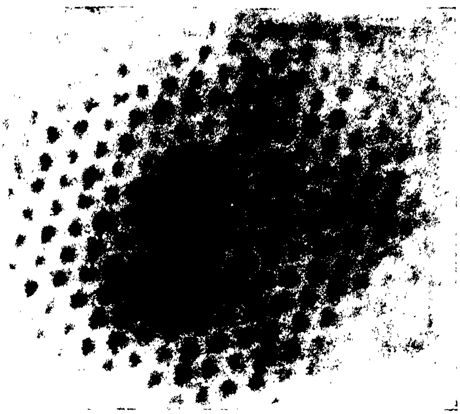


Fig. 43. Diffraction of electrons by very thin mica (thickness 10^{-5} *cm.).*

[*to face page 142*]

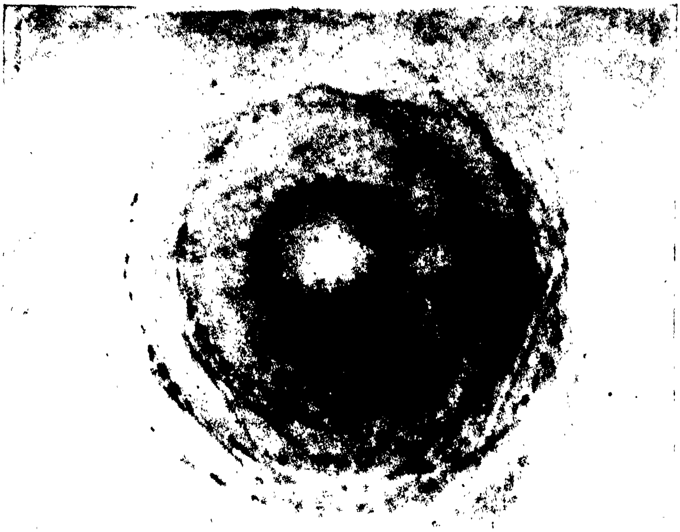


Fig. 44. Laue diagram, obtained by electron diffraction with mica plate 10^{-4} cm. thick (Kikuchi).

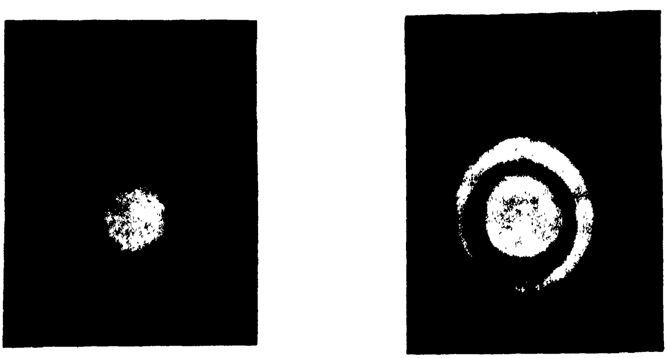


Fig. 45. Diffraction of cathode rays by celluloid (left) and gold foil (right) obtained by G. P. Thomson.

[*to face page 143*]

the middle or by cylindrical coils carrying an electric current, and therefore acting as magnets. This arrangement enables us to obtain in two stages a magnified image of objects through which these electron rays are sent; as much as 14,000-fold magnification is possible. The image is formed upon a fluorescent screen which shines where the electrons fall upon it. Other and even greater prospects are opening up. The magnification of an ordinary microscope is naturally limited by the fact that objects as small as the wave-length of the light used can no longer be seen sharply; but since the wave-length of electrons can be made very much smaller than that of visible light, by a suitable increase in their velocity, it is quite imaginable that the apparatus may some day be perfected to a degree enabling us to see very much smaller objects, and perhaps even molecules themselves.

But let us now return to our physical world picture. Here the prospects of a solution are worse than ever. All prophecy is useless. It is impossible to say how science will develop further. A new and brilliant idea is leading us along an entirely unforeseen road. We set out to find a way of reconciling two contradictory views of the nature of light. Instead, we have found that matter also seems to possess the same two-sided nature. When we are concerned with determining the path and the spatial position of moving matter, we must, strictly speaking, treat it as a wave phenomenon. Our only consolation is that our dilemma is not peculiar to light, but extends also to matter. Its existence is therefore in some way conditioned by the new constant of action h. And since this constant is so very small, the wave nature of matter can be neglected when we are dealing with it in bulk. But the constant is not zero. It appears to tell us correctly where our classical ideas cease to be valid. They hold only so long as the de Broglie wave-length is small as compared with the dimensions of the motion. This is not true of the electron orbits. Hence classical mechanics cannot be applied to them. There is no more point in speaking of the position of an electron in the interior of an atom than in speaking of the tracks of the photons when passing through a narrow slit. The electron is stretched out as a wave along its whole orbit.

It cannot be denied that our description of how the electrons behave as waves in the atom is still inadequate. De Broglie supposed these waves to be moving in curved paths around circles, and he took no account in his calculation of the force exerted by the electrically charged nucleus upon the electric charge which must somehow exist in the electron wave. We must replace the de Broglie wave picture by another. The "ray mechanics" which deals with corpuscles travelling along rays must be replaced, as regards the interior of the atom, by a wave which extends over the whole space surrounding the nucleus. The frequency ν of these waves must be connected with the energy of the electrons by the equation $E = h\nu$, and the wave-length λ with the impulse p by the equation $\lambda = \frac{h}{p}$. Here the amount of the impulse is determined by the force radiating from the nucleus. By making this plausible assumption, Schrödinger obtains, from the energy equation which governs the motion of the corpuscles, his wave equation by a generalisation which cannot be logically founded, but is successful.

The waves of an undisturbed atom must be what are called "stationary" waves. They cannot be travelling waves, such as are formed when a stone is thrown in water, and ripples travel outward in ever widening circles. A travelling wave becomes a stationary wave when the pool into which the stone is thrown has walls, such as the sides of a bath. These reflect the waves when they fall upon them, and the reflected waves interfere with the oncoming wave. Such stationary waves are frequently formed in harbours. The crests and troughs remain fixed in position. We obtain more perfect stationary waves when we tie a long rope by one end to a fixed point, stretch it out, and shake the other end with a steady rhythm. We then find that certain points move up and down, forming what are called loops, while other points remain at rest, and are called nodes. The string of a violin when plucked also falls into stationary vibrations of this type. The fixed ends, and perhaps some points between them remain at rest as nodes, while other parts of the string are in continual motion, the strength of which varies but is greatest at the loops (Fig. 46).

When de Broglie had succeeded in explaining the separate definite energy states of an atom by the conception of stationary waves, Schrödinger set himself the larger task of

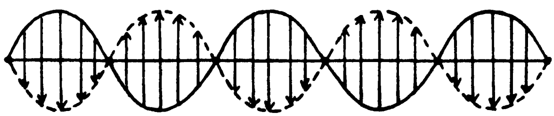


Fig. 46. Stationary wave. The points of maximum vibration, or loops, are at B, *and between them are the points of minimum movement, the nodes* K.

explaining, by means of stationary waves in space, all the various equilibrium states of an atom differing by whole quantum numbers, and also the changes in them produced by external influences. Stationary waves are always quantised even in classical physics. A given string is capable only of certain definite vibrations, as we see by the examples given in Fig. 47. The lengths of the waves are related to one another as $1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. The frequencies of the notes emitted by the string are therefore in the proportion 1 : 2 : 3 : 4. The three latter are called the "overtones" of the lowest note, the "fundamental". Other wave-lengths will not fit the string, and hence it is unable to emit notes of other frequencies.

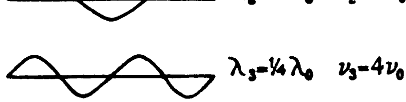


Fig. 47. Fundamental (λ_0, ν_0), *and first, second and third overtones of a vibrating string.*

Its notes are quantised, that is to say, are distinguished by a set of whole numbers. But it is not only stretched strings that behave in this way. Plates, set in vibration by means of a violin bow, also emit single definite notes (Figs. 48*a* and 48*b*). Each note is characterised by certain lines on the plate which remain at rest, the nodal lines, while the parts of the plate between them vibrate rapidly, as loops. If we strew sand upon the plate, it is shaken towards the nodes, and this

enables us to render visible the wave pattern. Wine glasses, especially of the broad and shallow type, may be filled with water and then forced to vibrate by stroking with a wet finger or a violin bow. The stationary waves are then exhibited as a pattern of waves on the surface of the water.

Such stationary waves are also possible in vibrating three-dimensional bodies, such as, for example, a sphere filled with liquid. We are to imagine a hydrogen atom as a stationary wave of this type in three dimensions. But how are we to imagine the wall enclosing this electron wave which is spread out on all sides around the nucleus? Where is the boundary which, by reflecting the wave, causes the formation

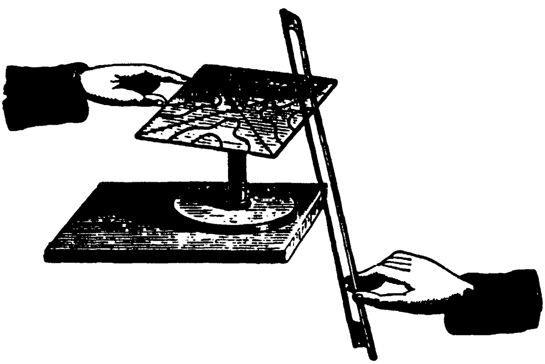


Fig. 48a. Vibration of plate produced by bowing.

of stationary waves with nodes and loops? In the case of the atom, there is no boundary at any finite distance from the nucleus; the wave travels over the whole of space, and hence an atom must reach out to infinity, where we must suppose the wave to be reflected. An obvious or natural boundary is given by the fact that the wave rapidly falls off as we go away from the nucleus, and disappears completely at a greater distance. A further natural condition is that proposed by de Broglie, namely that the waves should everywhere be of one kind (Fig. 42). Calculation then shows that only certain values are possible for the frequency of the waves, as in the case of the vibrating string. These values are commonly termed the "eigen" values, from the German word for "own"

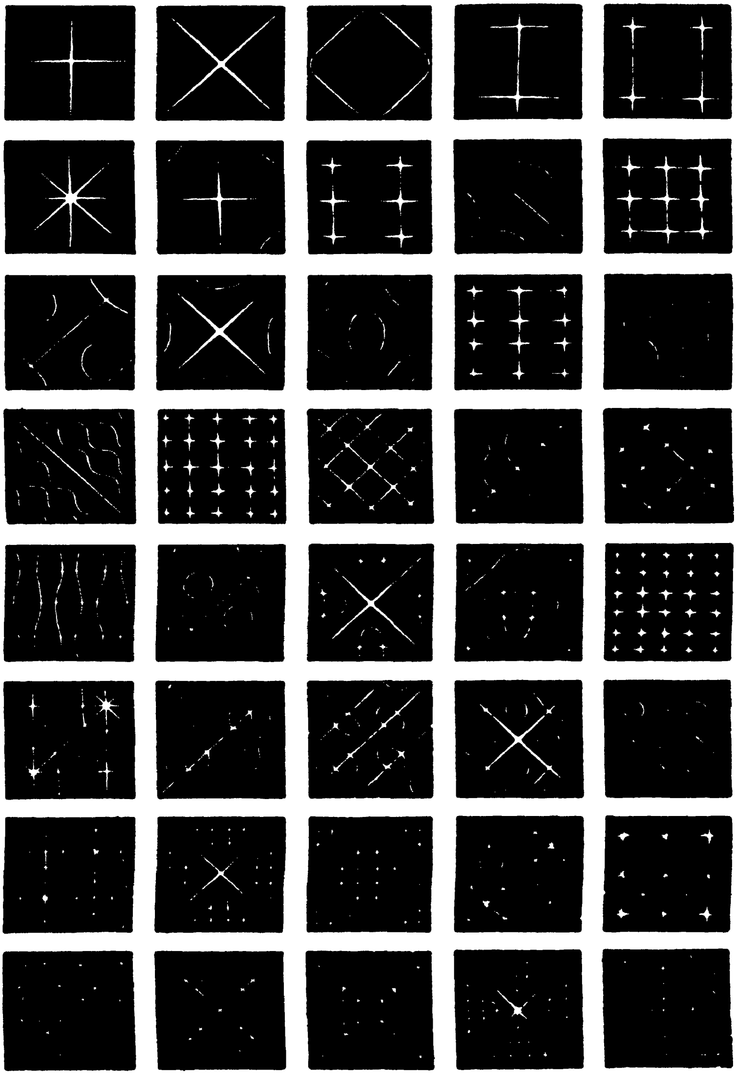


Fig. 48b. Nodal lines of vibrating plates.

used by Schrödinger. But since $E = h\nu$, this classical quantisation of the frequency also carries with it a quantisation of the energy. Hence, only definite eigenvalues of the energy are possible. Only quite definite energy levels can exist, which are characterised by whole numbers. The question now is whether these eigenvalues of the energy are those definitely known to us by our spectroscopic observation as actually existing. Are they the same as those as which Bohr's theory allowed us to calculate at the cost of accepting certain quite incomprehensible postulates? Schrödinger's famous essay on this subject was entitled "Quantisierung als Eigenwertproblem" (Quantisation as a problem of eigenvalues).

The calculations are not simple, but they are very necessary to the physicist. Only if the calculations result in correct energy levels, can we ascribe to the notion of matter as a stationary wave a greater amount of truth than the earlier atomic model. The decisive and important result of Schrödinger's calculation is that the possible energy levels of the hydrogen atom can be worked out without any further additional postulate. The values already determined by Balmer, and the correct value of the Rydberg constant, are found.

We do not yet know what it is that vibrates in these waves, but this vibrating something is limited to quite definite states of vibration in space, with a quite definite distribution of nodal surfaces. We here have the spatial counterpart to the sound waves possible to a vibrating plate as seen in Fig. 48*b*. To each state of vibration there belongs a definite value of the frequency, and hence also of the energy E. The various permitted energy values $E_1, E_2, E_3, \ldots$ differ from one another by a whole number $n = 1, 2, 3, \ldots$, the principal quantum number. But each of these eigenvalues, so it appears, can be realised by a whole series of states of vibration, which differ from one another because the nucleus is surrounded by various numbers of separate spheres, which are nodes of the vibration. Their number is found to be $n - l - 1$, where l is a new whole number which characterises these states. One state of oscillation only has no nodal sphere, and the strength of the wave in its case decreases continuously outwards without ever

vanishing. For this state, we have $n-l-1=0$ and therefore $l=n-1$. For the state with a single nodal sphere we have, since $n-l-1=1$, the value $l=n-2$.

Hereafter follow states with increasing numbers of nodal spheres, and values for the new quantum number $l=n-3$, $n-4$, ...; the extreme value is $l=0$. This is what we formerly knew as the azimuthal quantum number. Bohr's theory was unable to give us any reason why one had to count it with l from 0 to $n-1$, instead of with k from 1 to n; nevertheless the former mode of counting alone gave a correct result for the spectra of the higher atoms. Schrödinger's theory gives us the correct value without any such difficulty.

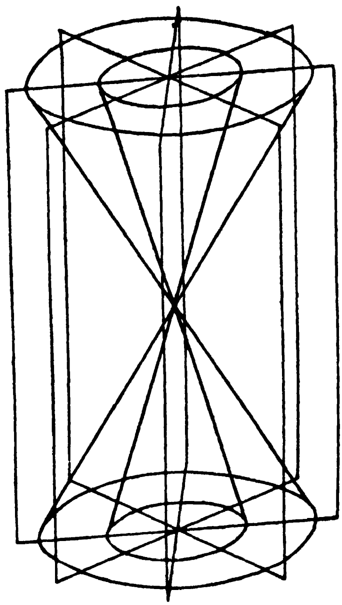


Fig. 49. Nodal planes and nodal cones of the hydrogen atom around the nucleus, which is the middle point of the nodal spheres (not shown).

Each of the eigen vibrations mentioned can further be realised in several, namely $2l+1$ different ways, which in general represent the same energy value. They differ from one another by the number of new nodal surfaces, namely nodal planes, and nodal double cones around the nucleus, which form the middle point of the nodal spheres (Fig. 49).

If we apply an external magnetic field, the new force throws all oscillations present a little out of tune, just as a vibrating violin string changes its pitch when touched by the finger. But when the frequency is changed, the energies are also changed a little, that of each vibration changing by a different amount. The $2l+1$ states of vibration (characterised by the magnetic quantum number $m=-1$ up to $m=+1$), which previously had the same amount of energy, now have slightly different values. The splitting up of the spectral lines can be calculated in a way which agrees exactly with experi-

ment. Schrödinger's theory can also explain the normal Zeeman effect without further assumption, and the same is true of the Stark effect, the splitting up of the lines in an external electric field.

The calculation of the energy levels of the hydrogen atom also gives us a further result. Besides the discontinuous series E_1, E_2, ... of possible eigenvalues of the energy, we find that from a certain upper limit of energy, which represents the loosest binding of the electron wave to the nucleus, all larger energy values are possible for the electron. The quantisation of the energy stops. While the values of the energy determined by quanta are those of an electron which has been captured by the hydrogen nucleus, the continuous succession of larger energy values belongs to all such electrons as have left the nucleus with this greater energy. These are free electrons. They are also waves, but their frequency and energy are no longer limited by quantum conditions.

Schrödinger's wave model of the atom contrasts favourably with Bohr's inasmuch as it is not flat, but in three dimensions. It is less concretely picturable, particularly as long as we have no idea what it is that vibrates as a wave. But this economy in concreteness results in greater truth to reality, as is shown by its greater power. Without making any additional assumptions, the wave hypothesis enables us to calculate all the conditions for the equilibrium states of the hydrogen atom. We shall also see that Schrödinger's model is superior to Bohr's in the case of the higher atoms. There are now two important questions which must be discussed. What sort of waves are these? What is it that vibrates in stationary waves about the atomic nucleus and results in what we call matter? When we have found good reason for a particular assumption on this point, we shall proceed to ask: How will these atoms consisting of matter waves behave when light falls upon them? Should they absorb and emit light in the manner demanded by the results of spectrum analysis?

CHAPTER VII

The Interpretation of Matter Waves

. . . the liberation thus won we owe to the wonderful progress of our insight into natural phenomena, a progress made during the last generation, and exceeding all hopes that we dared to entertain a few years ago.

—NIELS BOHR.

We have now become acquainted with two theories differing entirely from one another, and yet both of them equally superior to the original Bohr theory: the unpicturable matrix mechanics of Heisenberg, and the wave mechanics of Schrödinger. An important step forward was made when Schrödinger proved that the two theories are mathematically equivalent. But even without the use of mathematics, we may put the matter as follows. Both theories are agreed in rejecting the introduction, as fundamental concepts, of the position in space and the impulse of bodies, derived from classical mechanics. It can be proved mathematically that the essential features of the two, Heisenberg's peculiar rule for multiplication, which we have mentioned, and Schrödinger's replacement of ray mechanics by wave mechanics, are equivalent. Furthermore, we are able to state exactly which mathematical expressions in the two theories correspond with one another. But since this cannot be understood without mathematics, we will not attempt an explanation, but simply take it as a fact. Only one thing more must be said. In Schrödinger's theory we are dealing with a something that is vibrating as a wave. We have as yet no indication of the nature of this something, but we know what it does. It vibrates in waves. In order to know what states of equilibrium the hydrogen atom can assume, we do not need to know, as we have seen, the subject of this activity. The same fact emerges in matrix mechanics. This is a system of rules for calculation of which we are not able to say to what

they are to be applied. Apparently, it is the mathematical operations which are the important point, and not the something to which they are applied. There exists something in matter which has the character of a wave, but we do not know what it is that vibrates. This, by the way, is nothing new in physics. We have already come across it in this very case of waves. Physics had long ago developed its conception of the wave nature of light before there was any agreement as to what it was that was in vibration. When the belief in the mechanical vibration of a material ether was dropped, and the electromagnetic nature of the waves was recognised, the results of the mechanical wave theory, such for example as the diffraction of light by a fine grating, still retained their validity, even when the subject of the vibration was changed. The knowledge that something is in vibration already represents a step forward.

But we certainly know more when we know what it is that vibrates. Schrödinger himself has proposed an interpretation of these waves. One thing is certain; it cannot be any kind of matter that is vibrating, for the corpuscles of matter, the electrons, are actually constituted out of waves in some other natural medium. Schrödinger assumes that the waves measure by their strength the density of the electric charge possessed by the matter, the electron. This charge is therefore no longer to be imagined as concentrated in the body of the electron, but rather as distributed over the whole wave structure, which is extended without limit, but falls off rapidly as we go from the electron, like the sand on the vibrating plate. The stronger the vibration at any point, the denser is the electric charge there. It is densest at the loops, and least at the points of least motion, the nodal surfaces. This is the reverse of the density of the sand spread upon the vibrating plate, since this collects along the nodal lines. We no longer have corpuscles, they are now resolved into a "charge cloud". This hypothesis is attractive in its simplicity, and in the unity with which it comprehends all atomic phenomena.

Let us for the moment ignore possible objections, and again ask what the hypothesis can do. Schrödinger uses the method familar to physicists for dealing mathematically with wave

processes. We will attempt to get some idea of the range of Schrödinger's hypothesis, by making use of the concrete bases of this calculation, as we meet them in acoustics.

We need to use two main concepts: resonance, and beats. If we put two musical instruments, say two tuning forks, tuned to the same frequency and therefore emitting the same note, side by side, and cause one to sound, the other also commences to vibrate, since it is struck by the sound wave emitted by the fork first set in vibration, and transmitted to the second through the air. This happens because the waves strike the second fork rhythmically, at intervals which correspond exactly to its own natural rhythm. The impulses of the air wave each come at the right moment to reinforce the vibration of the second fork; each push is rightly timed to increase the swing of the fork. This is called resonance. But if the second tuning fork has a slightly different natural frequency, it is now no longer struck in the correct rhythm by the waves emitted by the first; it no longer responds to the blow. It emits no sound, for it is not in resonance with the first. The window pane which buzzes when a certain note is struck on the piano, is in resonance with that note. We are familiar with the fact that a wireless receiver must be tuned to be in resonance with the frequency of the electric wave of the station which we wish to receive.

Now we must consider beats. If we set vibrating two strings which are slightly out of tune with one another, let us say to the extent that one makes just one more vibration in a second than the other, they will be vibrating together at the end of a second if they were doing so at the beginning. If the first has a frequency $\nu = 435$, then the other will have $\nu = 434$. At the beginning and end of a second they will reinforce one another, but in the middle they will vibrate oppositely; the crest of one vibration will exactly coincide with the trough of the other, and they will therefore destroy one another. Hence the sound falls to zero in half a second, and reaches full strength again at the end of the second. The same effect is repeated in every following second. We hear the note rise and fall alternately, and perceive the effect known as beats. If the frequency difference is two per second, we have

in every second two ups and downs of loudness, that is, two beats. Quite generally, the number of beats n is equal to the difference in frequency of the two vibrations:

$$n = \nu_1 - \nu_2 .$$

If the number of beat impulses n becomes so great that they cannot be counted separately, they affect the ear like a new note of a much deeper pitch than either of the two notes which are beating together. This is the so-called difference note. If we carefully press down the C on a piano so as not to strike the note, and while holding it down strike C and G together in the next octave above, we then hear the lower C sound. It is given by the equation $\nu_1 - \nu_2 = \nu_3$ (Fig. 50). We can also hear a summation note, that is to say, the note of frequency $\nu_4 = \nu_1 + \nu_2$, in our case the third above. This note is usually much weaker. After this digression into the physics of sound we are now in a position to understand how we are to imagine, according to Schrödinger's hypothesis, the interaction between an atom and light.

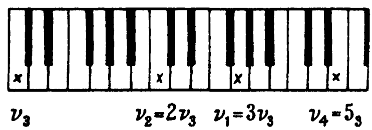


Fig. 50. The difference note $\nu_3 = \nu_1 - \nu_2$ and the summation note $\nu_4 = \nu_1 + \nu_2$.

If the atom is not influenced from outside, it vibrates, in a manner determined by the field of force of its nucleus, as a stationary wave of one of the frequencies ν_1, ν_2, ν_3 ..., as a rule, presumably, in the most stable, lowest frequency, ν_1, with the smallest energy content E_1. Our electron corpuscles have disappeared altogether. Their electric charge is distributed like a cloud with its smallest density at the nodal surfaces, and greatest density at the loops. This distribution continues as long as no influence is exerted from outside. But as long as the electric charge distribution is unchanged, there is no radiation according to classical principles. The atom is in a state of equilibrium. This represents a step forward in our point of view. Rutherford's model of the atom allowed of no understanding of the normal non-radiating state of an atom on the basis of classical physics. Bohr produced his peculiar table of prohibitions for his electron

corpuscles, which accounted in a way for the absence of radiation; but matter in the form of a standing wave retains its loops and nodes, that is to say, the distribution of electric charge.

If a light wave now strikes the atom, it will, as we know, be as a rule partly reflected, and partly transmitted, without being absorbed. Only light of quite definite frequency is absorbed, namely that which fits Bohr's frequency condition. Is our wave model able to explain this fact, experimentally fully confirmed, but hitherto quite incomprehensible? Let the frequency of the oncoming wave be ν_2. We can then combine the frequency ν_1 of the atomic wave with the frequency ν_2 as we can combine the frequencies of piano strings vibrating together, to form a summation or a difference tone. If the atom was previously vibrating with its fundamental frequency, it cannot go over to a mode of vibration of lower frequency, but it can assume the higher frequency of a summation tone. The atom is like a piano all the keys of which are pressed down so as to take off the dampers: like the piano, the atom has a definite limited number of possibilities of vibration. When the resulting summation tone $\nu_1+\nu_2=\nu_4$ is a possible note ν_4 of our atomic piano, then and then only will it respond to this new note, that is to say, go over into the new state of vibration. Generally speaking, therefore, light striking an atom will pass through it, and only single definite colours, such as the frequency $\nu_2=\nu_4-\nu_1$, will influence the atom. They will put the atom into an "excited" state.

In our earlier corpuscular language, we spoke of the energy of electrons, instead of the frequency of waves. If E_1 and E_4 are the values of the energy in the normal and the excited condition, we have $E_1=h\nu_1$ and $E_4=h\nu_4$, from which it follows, since $\nu_2=\nu_4-\nu_1$, that

$$\nu_2=\frac{E_4}{h}-\frac{E_1}{h} \qquad h\nu_2=E_4-E_1.$$

The atom can therefore only absorb those wave-lengths of light which correspond to such conditions. This is none other than Bohr's frequency postulate. It follows therefore quite

naturally, without further assumption, from Schrödinger's hypothesis.

If the oncoming light is not monochromatic, but white—that is to say, if it consists of a large number of different colours with very different frequencies—more than one colour will be absorbed: all those colours, namely, which can excite possible states of vibration. The light which has passed through the atom will exhibit the full absorption spectrum.

Now let the atom be vibrating in the "excited" frequency ν_4. It can then return to its fundamental frequency ν_1. As long as this is happening, it possesses simultaneously two different values of energy, $E_1 = h\nu_1$ and $E_4 = h\nu_4$. This is impossible as long as we regard an electron in the manner of classical physics, or even of Bohr's theory. A single electron cannot travel at the same time in two different orbits around the nucleus, but two strings of our atomic piano can certainly vibrate at the same moment. We then get, as in the case of the piano, beats between the two vibrations, and these have the frequency $\nu_4 - \nu_1 = \nu_2$. In the case previously considered, when the atom was vibrating in only one manner, the electric charge is unchanged in position in it, but this charge will now oscillate to and fro in the rhythm of the beat ν_2. But every wireless station emits waves of the same frequency as the alternating current of electricity in the antenna, and this shows us that an electric charge swinging to and fro rhythmically emits an electromagnetic wave according to classical principles. Our atom therefore emits light of the same frequency as that which it has just absorbed. It again follows from $\nu_2 = \nu_4 - \nu_1$, that $h\nu_2 = E_4 - E_1$, which is Bohr's frequency condition. When our wireless receiver is wrongly adjusted as regards reaction, it emits a wave, the frequency of which is close to that which we wish to receive, and we then get the unpleasant howl caused by beats between the incoming wave, and the wave of our own receiver. In exactly the same way, the beats of the atomic waves result in light waves.

The strength of the beats, and hence the brightness of individual lines of the spectrum, can also be calculated. The result is excellent agreement with experiment in all the very various cases in which the calculation has been made.

Certain lines should by calculation be so weak as to be invisible. The earlier theory required a "selection" principle which could not be given a logical basis, but was fully confirmed by experiment; this principle can now be exactly deduced by wave mechanics. Schrödinger's delight was well justified, when he said in a lecture given before the Royal Institution in 1928: "Of course it is impossible to set forth in this lecture any of the calculations that led to the results just given; they would fill pages and pages, and are not at all difficult, but *very* tedious. In spite of their tediousness, it is rather fascinating to see all the well known but not understood 'rules' come out one after the other as the result of *very* familiar elementary and absolutely cogent analysis, like e.g. the fact that $\int_0^{2\pi} \cos m\phi \cos n\phi \, d\phi$ vanishes unless $n = m$." We will not however carry out these "very elementary" calculations.

If the frequency ν_2 of the oncoming light is very high, that is to say in the ultra-violet or X-ray region, the summation frequency ν_4 resulting from its combination with the atomic frequency ν_1, may be so great that the corresponding energy, $E_4 = h\nu_4$, is no longer one of the single possible atomic energy values, but belongs to the continuous succession of energy states proper to electrons which cannot be held fast by the nucleus. In this case, a matter wave, that is to say, an electron, is emitted. Wave mechanics is thus also able to explain the phenomenon already known to us as the photoelectric effect, not only qualitatively, but quantitatively. The same is true of the Compton effect.

Let us now survey the result of our investigation. Schrödinger's hypothesis, which substitutes for the corpuscular electron an electron wave, is able to represent correctly, without additional assumptions, the whole known range of quantum phenomena. In some cases, by the way, it predicts slight differences from the original Bohr theory, and these have also been found by experiment to exist. The theory is thus a great advance on Bohr's. The discontinuity in atomic behaviour, which at first seemed so extraordinary, has been rendered comprehensible with the aid of classical concepts

such as the stationary wave. The radical pessimism expressed by Bohr and Heisenberg when the first theory failed, thus appears premature. But here, as always in science, there comes a *but*. This hypothesis of the nature of matter waves is not without some riddles of its own; one particularly puzzling question is the following.

The electric charge of the electron "cloud" is supposed to be distributed over the whole space occupied by the atom. Now this calculation of the cloudlike distribution of charge is based upon the law of classical physics, that electric charges separated in space influence one another by attraction or repulsion in quite a definite way. This law determines the influence of the positive charge of the atom, imagined as concentrated at a point in the nucleus, upon the external negative charge, but no notice whatever is taken of the fact that the separate parts of the negative cloud charge must also exert forces upon one another. If such forces are taken account of, we get wrong results.

Another objection is as follows. The electrons cannot always be waves, for such waves must be spread out in space, whereas the cathode rays undoubtedly consist of negative electric charges, each concentrated in a very small space, and obviously corpuscular in nature. Schrödinger has tried to defend his hypothesis against this objection. It is true that we cannot assume the freely moving electron in a cathode ray to be a single wave of definite frequency. Such a wave would spread out uniformly into the whole of space, and the electron would then fill all space and not be concentrated in a point. But the case is otherwise if we regard the electron as the point of overlapping of many waves. Let us first consider the effect of only two waves of different frequency travelling in a straight line and overlapping (Fig. 51). We get a beat wave with alternate high and low crests. If we now add a third wave of intermediate length, a few crests and troughs become more marked, while those in between are lessened by interference.

If we now add still more waves with intermediate frequencies, they all strengthen one another in the region around the initial point, since here they are all in the same

phase of vibration. On the other hand the high wave crests farther out no longer increase sensibly, since on account of the difference in wave lengths from wave to wave, the crests all fall at different positions, and the result is to flatten all the more distant regions. Only the region about the initial point is filled with a prominent crest. Also, a number of spherical waves spreading out as surfaces into space and differing from one another by small amounts, combine together to form a wave structure limited to a small region. According to Schrödinger, we are to imagine a corpuscle

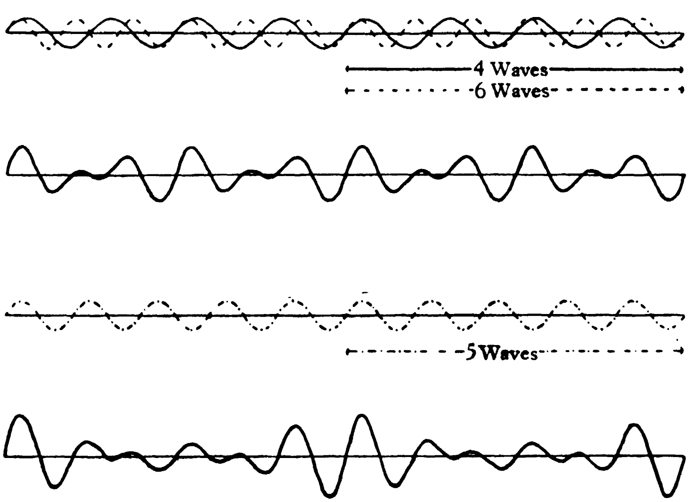


Fig. 51. Overlapping of waves.

such as a freely moving electron to be a "wave packet" of this kind. We thus appear to have arrived at the important result that the dualism in matter between corpuscles and waves is to be decided in favour of the latter. There are no such things as corpuscles: there are only "wave packets". Unfortunately, a closer examination of this idea shows us that these wave packets will not in general hold together permanently, as they must if we are to explain the stability of bodies, but would gradually be dissipated, although this process would be very slow in the case of bodies of ordinary size.

The dualism particle-wave is thus not abolished. Matter waves certainly exist, but they cannot be thought of as the concentration of electric charge. The concrete and picturable hypothesis that the wave loops are positions of high electric charge, and the nodes positions of least charge, cannot be carried through.

The waves which take the place of electrons are thus not a measure of the charge density; the corpuscles are not to be regarded as the points of greatest agitation of a mixture of waves. Matter waves exist, but it is still a riddle how the corpuscles are connected with the waves. Our problem remains unsolved: how can an electron, and also a light quantum or photon, spread out from a centre like a wave in all directions, and nevertheless reappear at every point of this wave front as a complete whole, as a corpuscle possessing full energy and full impulse? Why a wave can everywhere exert action like a body, and why on the other hand bodies do not travel as rays, but as waves, is still a complete mystery, as much in the case of light as of matter. The matter waves are not concentrated electric charge, nor the light waves the essence of light energy. Both waves must have a different meaning. It must be something else that moves in a wavelike manner.

There is still another decisive result which makes these waves of matter still more mysterious. Schrödinger's calculations show that, on the one hand, the hydrogen atom with its single electron is to be regarded as a wave in our familiar space of three dimensions: up and down, right and left, forwards and backwards. But if we are dealing with an atom possessing two electrons, the calculation calls upon us to regard the corresponding matter wave as existing in space of 2×3 dimensions. Space of this kind can no longer be imagined concretely; it cannot be the familiar space in which we make our observations. But we are able to calculate with six dimensions just as well as with three, and when we have atoms possessing three and more electrons, we can calculate with nine and more dimensions. Hence Schrödinger's waves are not to be imagined as existing in our ordinary space; that is only possible when we are considering a single electron. In

general, these waves are vibrating in spaces of higher dimensions. Schrödinger speaks of "configuration" space, which is an expression of the fact that the mathematical equations describing the behaviour of electrons are the same as those familiar to us with ordinary waves, excepting that in them appear more than three numbers referring to dimensions of space. We are no longer able to form a picture of such waves, for we can only picture things in three dimensions, but they are, nevertheless, real. For they enable us to answer any question dealing with the experimental results of any given distribution of matter and light. But we cannot go so far as to assume the existence in these abstract waves in higher dimensions of electric charge concentrated in the loops of the waves. They must have a more abstract, a symbolic meaning.

It almost appears a matter of chance that the waves ascribed to a single electron exist in only three dimensions. Nevertheless, this has been proved experimentally. But Heisenberg's point of view also leads us to regard matter waves in general as less concretely real. In his view, the wave constants are only mathematical expressions for magnitudes occurring in quantum mechanics, and hence must share the want of concreteness of the matrix system. These matter waves are not therefore something we can perceive with our senses, but simply a mathematical abstraction. Something exists in the world "as if" it were waves. Something in the form of the micro-world can be described by means of the wave as a symbol. Wave mechanics from this point of view is a piece of good luck, since it enabled Schrödinger to devise a mathematical instrument much easier to use than matrix mechanics and nevertheless leading to a logically perfect formulation of the laws of the atomic world. The fact that it also enables our imagination to form a certain picture of events must also be regarded as an advantage; but the hope that wave mechanics would reduce quantum phenomena to the terms of classical physics, without the need for any sacrifices being made, has proved a delusion, though the attempt was a magnificent one. As a result of all this, physicists are gradually beginning to doubt whether ideas

derived from the everyday world can ever enable us to understand the world of atoms.

But whatever the course of further development, whatever may be the meaning of these waves, we have gained something which is independent of their interpretation, namely, the power to calculate the eigenvalues, the frequencies, and the intensities of spectral lines. Mathematical methods have frequently proved in physics to be more durable than the interpretation which they have from time to time been given.

We have therefore certainly not yet discovered the whole truth, perhaps not even the most important point; but we shall never discover *the* truth. We never come upon it all at once. We pass on from error to error, and are delighted when the new error is less than the previous error, and we feel that we are gradually approaching the truth, although the road is endless. In research there is always an admixture of misunderstanding; in the best case, it is the misunderstanding of genius; this is not only true in the moral sciences, but in physical science as well. Natural science perhaps has the advantage that its road does not wind too much, but is fairly straightforward. It leads us, at least in the belief of physicists who do not deny metaphysics altogether, in the direction of the world of "things in themselves", which we believe to lie behind the world of experience, though it be for ever a sealed book to us.

Classical physics had discovered part of the truth. With the first quantum theory, physicists delved more deeply, and discovered new grains of truth. Wave mechanics, in the form which we have hitherto discussed, then revealed a new bright nugget of truth, but here again the noble metal was embedded in dross. We must dig still more deeply if we are to win it in a purer form.

Let us turn to the next chapter of this exciting romance of modern physics. We have realised that wave mechanics has not succeeded in overcoming the dualism of particle and wave, which affects both atoms and light. It has been found necessary to ascribe wave properties to matter, but we must not therefore conclude that matter does not possess the concrete properties hitherto ascribed to it. For our views of

light have suffered a similar change. According to Newton, it consisted of material particles. Further investigation showed that the manner in which light spreads out into space and past obstacles can be better described by regarding it, not as particles travelling along rays, but as waves, which, for example, can be diffracted by narrow slits. Only when we are dealing with regions large compared with the wave-length can we in every case regard light as particles travelling in rays. But wave optics, however accurately it describes the motion of light in space and time, does not give a correct picture of what happens when light is emitted or absorbed. The exchange of energy and impulse which then takes place can only be understood by adopting the point of view of the light-quantum hypothesis, and regarding light as consisting of particles, the photons.

The advance made by Huygens in the interpretation of light corresponds to the new view of matter which we owe to de Broglie and Schrödinger. The motion of matter in space and time must be regarded from the point of view of wave mechanics. This means that we can only speak with sufficient accuracy of bodies as moving in well defined paths when we are dealing with motion in dimensions large compared with the wave-length of matter, that is to say, with problems of everyday mechanics. But wave mechanics, which confines itself to the wave concept, does not suffice to represent correctly the energy and impulse transformations of matter. Here also we are unable to do without the concept of "body".

Both attributes, body and wave, possess reality, but neither can replace the other. The light emitted by an atom appears at its birth as a photon, possessing a certain quantity of impulse. It moves, as must be imagined when it passes through a narrow slit, as a wave possessing a definite frequency and length. Somewhere or other, it is again swallowed up by other atoms, and the effect is as if it again consists of photons of a definite energy and impulse. And precisely the same is true of very small portions of matter. The electrons in a cathode ray tube are shot as minute material bodies out of the cathode, and their motion over long distances is like-

wise to be described as the path of such bodies or corpuscles. But when they pass through narrow openings, as in a crystal, or are captured by an atom, they are compelled to move within its minute limits, whereupon they cease to be corpuscles, but become waves, of the nature of which we can hardly form any concrete notion, for they seem to exist in space of higher dimensions. On the other hand, if they strike other atoms, they again appear to be minute bodies carrying energy and impulse.

This extraordinary use of two contradictory notions requires an explanation. We must be making some assumption which appears self-evident, but is wrong all the same.

The first successful step towards a solution was made in 1926, by Max Born. He proposed a new hypothesis to explain the double nature of radiation and matter. Let us first consider light. In the wave theory, the height of the wave crest had always been regarded as a measure of the brightness of the light. But if light is regarded as consisting of photons, greater brightness would mean the arrival of a greater number of photons at the particular illuminated point considered. If photons and waves both exist, the waves must measure by the height of their crests the number of photons present. This is the new hypothesis concerning the meaning of waves.

Photons are emitted by a luminous body. While they are passing through space, we speak of them as light waves. Light waves spread out, perhaps interfere with one another or other waves, are diffracted round small obstacles, and somewhere or other strike a screen. At this screen, a number of waves will strengthen one another at many points by interference, and at these points we have high wave crests. We conclude, according to Born, that here numerous photons are arriving. This part of the screen will be brightly illuminated by a plentiful supply of luminous energy. At other points, the light waves extinguish one another by interference, and here we do not expect any photons. At such points on the screen we have darkness. No photons have been destroyed, but the *probability* of any arriving at these points is very small. The wave crest indicates the statistical average value, the probable

number of photons arriving. What travels outward in waves is this probability of photon arrival. In this theory, the photo-electric and Compton effects agree with the diffraction experiments. It is always photons that move. When they meet with matter, they make their presence perceptible by their energy and impulse. But the manner in which they spread out in space and time is, strange to relate, not given by the laws of motion otherwise familiar to us in connection with bodies; the motion is indicated by waves. These move strictly according to the laws of the classical wave theory. The fate of a single photon on its journey, its path as we say, remains quite undetermined; such paths do not exist, the motion is given by waves, and these determine the average number of photons which arrive at any point. We can only make statistical statements concerning photons.

We now apply the same hypothesis to matter. It also consists of bodies, the electrons and protons. But these bodies also are not the bodies of ordinary physics, but have a wave character. The matter wave measures by the height of its crest the probability of the arrival or presence of the particle. This assumption explains electron diffraction in a satisfactory manner. Waves can strengthen and also extinguish one another. At one point, many electrons arrive, at others few. The waves are again "probability waves", and their course is determined by the laws of wave motion given by classical physics. Also, only statistical statements can be made concerning the particles of matter. The motion of individual particles remains quite undetermined. Their case is just as curious as that of photons. From this point of view, we understand Heisenberg's form of quantum mechanics, in which no mention is made of motion.

The theory tells us nothing of how the position and velocity of a particle change, and it also does not reveal why we are unable to determine them; whether the cause of our ignorance is insufficient knowledge, which may somehow or other be overcome, or whether we are confronted with a fundamental impossibility. Let us put this problem aside for the present, and note that the Born theory at least renders it possible to use together both concepts, particle and wave. We

recognise at the same time that one difficulty of Schrödinger's hypothesis no longer exists. These waves which measure the "probability distribution" of the electrons can also be given more than three dimensions in space; for they are waves of a formal mathematical character.

A freely moving electron subjected to no forces possesses a matter wave spread out over the whole of space. This no longer means that a single electron is itself spread out over all space. Our theory is statistical, and makes no statements concerning the motion of a single definite electron, but simply concerning the average behaviour of a great number. In the present case it says: "If we have a large number of electron points able to move freely, the probability of finding one of them is equal all over space."

In the atom, the loops of the stationary waves are the densest points of the average distribution of one or more electrons, and here again the statistical calculation applies to a large number of atoms. Bohr's electron orbits now recover a certain degree of reality. The radii of the Bohr orbits in the hydrogen atom are approximately equal to the most probable distance of the electron from the nucleus.

In other respects the theory is much more abstract. Schrödinger's original interpretation provided a very concrete explanation of Bohr's frequency postulate. The matter waves represented by the atom need only be imagined as possessing two different frequencies at once, which must result in beats. The electric charge vibrating to and fro in a definite rhythm necessarily produces a light wave of the same frequency, and this frequency is exactly that called for by Bohr's condition. We now have to imagine the radiated light wave as a beat wave resulting from two waves giving the probable distribution of the electrons imagined as corpuscles. This is a much more abstract point of view. But the rule itself, and also the other conclusions drawn mathematically from the wave equations, and agreeing so excellently with experiment, will not be given up, but rather allowed to await a better interpretation as a result of deeper insight into the nature of Born's hypothesis. For it is not the more or less concrete pictures which we are able to associate with a

theory, but its mathematical content, which is the essential feature of it. Born's hypothesis forms a remarkable reconciliation between the particle and the wave aspects of matter and light. Both views contained a portion of the truth. Light and matter are not waves, and they are also not bodies. They behave as if they were bodies whenever we are concerned with an exchange of energy and impulse. But something essential to reality is wanting for these corpuscles. It is impossible to ascribe to them a definite position, or a definite path in which they move, for their average motion in space and time is such as if they were waves. Both conceptions have something schematical and dead.

The concept of the atom as the indivisible smallest constituent of matter, nevertheless possessing extension in space, has always contained a contradiction. In imagination, it can always be divided into smaller particles. And the atoms are in fact divisible. The same difficulties then apply to the new smallest particles, the electrons and protons. But this difficulty is now abolished in the new point of view; the smallest particles are not material in the sense derived from our impressions of ordinary bodies. I can imagine a tiny lump of matter in the ordinary sense as being further divisible, but not so mysterious a thing as an electron. It no longer possesses all the attributes of a macroscopic body. Here we come upon something absolutely new. In the world accessible to our senses, there are things which we fully understand physically when we say that they are bodies, and others, which in the same way are waves. In the world of the atoms, these two notions both fail us; both are necessary, and each alone is inadequate at some point. Planck's constant h is the new magnitude which marks the transition from the corpuscular magnitudes, energy and impulse, to the wave characteristics, frequency and wave-length, in the equations

$$E=h\nu \quad \text{and} \quad p=\frac{h}{\lambda}.$$

Neither point of view can be carried through alone, as we can see by the following argument. We know that an electron moving freely is on the one hand a small body,

possessing a perfectly determinate energy E, and an equally determinate impulse p. On the other hand, its position is represented by a wave filling all space, possessing frequency ν and wave-length λ, which can be calculated by the above equations from the values of E and p. These exactly known values of E and p for an electron thus correspond to a complete uncertainty regarding the point at which it is present at any given time. Only when we cause several waves differing in wave-length over a certain range to overlap do we get, as we have seen, a wave possessing perceptible height in a definite region of space, and disappearing in the whole of the rest of space. Only such overlapping of several waves of different wave-length produces a "wave packet". These are waves concentrated in space which travel forwards together as long as they hold together, and behave, therefore, like a particle.

The smaller the region occupied by a wave packet and the more definite its path, the greater the number of different waves which go to form the packet by overlapping. As the difference between their λ's and ν's increases, that is to say, as the mean values of λ and ν become less and less definite, the uncertainty of our knowledge of E and p also increases, according to the two fundamental equations. As our knowledge of the values of E and p gets less and less accurate however, our ability to treat the wave packet as a particle increases; we can ascribe to it the localisation in space of a particle moving in a path according to classical law. The mistaken feature of Bohr's original conception lay in imagining the electrons as travelling in definite orbits at a definite speed, thereby making definite statements concerning their position and times of rotation, while at the same time ascribing to them, in the postulates, quite definite values of energy and impulse. More than twelve years of work by Bohr and a whole army of other physicists, was necessary to render evident the fact that such statements can only be made in macrophysics. In quantum mechanics, which sets out to explain micro-events, they must not be made. The two pairs of magnitudes, position and velocity, energy and impulse, can never be both stated accurately at the same time of any given body.

This statement must be now justified. Let us recollect the phenomena of beats occurring between vibrations. If we wish to tune two strings of an instrument accurately together, we can do so by causing them both to sound at the same time. Suppose they are a little out of tune. One, let us say, vibrates with $\nu_1 = 450$ vibrations a second, the other makes in the same time $\nu_2 = 453$ vibrations. The difference of frequency $\nu_2 - \nu_1 = 3$ can be found by determining the number of beats in a second between them, but it is necessary to listen for a whole second, or at the very least, it is necessary to measure a time

$$T = \frac{1}{\nu_2 - \nu_1} = \frac{1}{3} \text{ second},$$

that is to say, the duration of a single beat. A shorter time of observation would not enable us to decide by how many vibrations the two notes differ. The difference between the frequencies of two notes $(\nu_2 - \nu_1)$, and the time T which is necessary to determine this difference, are always related by the equation

$$(\nu_2 - \nu_1)\, T = 1.$$

The smaller the interval between the two notes the longer is it necessary to listen in order to determine it.

In the above case we have considered the variation with *time* of the intensity of a wave. Let us now consider the case of a diffraction pattern formed on a screen, such as the interference bands produced by a grating. These represent a succession of varying wave intensities in *space*, such as are found in every diffraction image. Let us follow the course of two light waves proceeding from two points separated by a small distance a, say two neighbouring points of an optical grating (cf. Fig. 18, p. 37). This distance can be determined by measuring the interference bands, say the distance apart of two neighbouring bright points b, and the distance c between the grating and the screen on which they are projected. These are related, as we know (p. 36), to the wave-length λ of the light by the relation $b : c = \lambda : a$. From this we can easily calculate the spacing a of the grating, when the other values are known. In our example (p. 37), the wave-length of the

red light was 0·00007 cm., the distance b on the screen was 16 cm., and c, the distance between grating and screen, was 560 cm. From this we deduce that

$$a=\frac{0{\cdot}00007\times 560}{16}=\frac{1}{400}\text{ cm.}$$

In every case $$\frac{ab}{c}=\lambda.$$

In order to be able to measure what might be called a beat in space, that is the distance apart of two bright bands, we must be able to see a part of the screen subtending an angle given by the fraction $\frac{16}{560}$ (Figs. 16–18). Only then is it possible to determine the distance apart a of the two sources of light. The case is exactly analogous to the preceding case; the smaller the distance apart of the sources, the farther apart are the interference bands, and the greater is the angular opening within which we must follow the path of the light.

Both equations are true of all kinds of waves, not only light and sound waves, but also matter waves. We now introduce the important new fact that both light and matter waves have also the properties of particles, the characteristics of which, namely energy E and impulse p, can be calculated from the wave characteristics ν and λ by means of the equations given by Planck and de Broglie.

$$E=h\nu,\qquad p=\frac{h}{\lambda}.$$

We then can form the following conclusions concerning the corpuscles of light and matter. From $(\nu_2-\nu_1)\,T=1$, it follows by multiplication with h that $(E_2-E_1)\,T=h$. Here $E_2-E_1=e$, the difference in energy between two small bodies. The equation

$$eT=h,$$

then states a quite extraordinary fact; if we wish to determine the energy difference e between two corpuscles, a certain minimum time of observation T is necessary, which is given by the above equation. We may also say: in order to determine the energy of a single corpuscle with an error not

greater than e, we require a minimum time of observation T. The more exactly the energy is to be determined, the smaller in other words is to be the error e in the energy measurement, the longer must be the time of observation T. Since h is an extremely small quantity, the energy measurement, if the required accuracy is not excessive, will require only a very small time of observation. This is the case in the physics of ordinary bodies. We then are not conscious of any definite minimum time, and we regard the energy of a body as some-

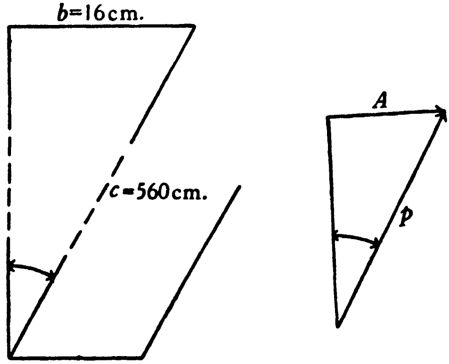


Fig. 52. b = *distance between two diffraction maxima. From the brightly illuminated centre to the neighbouring dark positions, the distance in both directions* $= \frac{b}{2}$, *and* $b = \frac{b}{2} + \frac{b}{2}$. A = *degree of uncertainty with which the sideways impulse of the moving particles is known.*

thing that simply exists, and is independent of any time factor.

Let us now consider the other equation $a \frac{b}{c} = \lambda$. This can be transformed by using $\lambda = \frac{h}{p}$ into $a \frac{b}{c} = \frac{h}{p}$ or $a \left(p \frac{b}{c}\right) = h$. Its significance can easily be perceived with the aid of a simple geometrical argument (Fig. 52). $\frac{b}{c}$ measures the angle subtended by the distance of the first diffraction maximum from the centre. p stands for the impulse of the flying corpuscles belonging to the wave. Since the corpuscles belonging to the angular opening considered may meet the screen anywhere

over the whole distance b cm., their impulse is undetermined. Some corpuscles appear to have travelled straight forward, others to have moved sideways. p denotes the direction of the next diffraction maximum. There is thus an uncertainty regarding the impulse just as there is regarding the energy. Let the amount of it be A. We see from the figure that

$$A : p = b : c;$$

hence
$$A = p\,\frac{b}{c}.$$

Our equation thus becomes

$$aA = h.$$

This means, that if the position is known at which a flying particle was present within the limits given by the small distance a in a horizontal direction, then its impulse, likewise in a horizontal direction, is uncertain within the limits of the small error A. The same is true of all other directions as well. Hence the more accurately we wish to state the impulse, and hence the velocity of a body, that is to say, the smaller A is to be, the greater is the error a made in the determination of its position, and the less exactly is its position known. The converse is also true. This fact does not appear in ordinary physics on account of the smallness of h.

This investigation of the possibility of measuring fundamental mechanical quantities, has thus led us to the same result. Classical physics was wrong in believing that bodies existed to which a definite value of energy, and hence impulse and velocity, could be ascribed, without at the same time their position and the length of time that they were under observation being known. This is not true in the small dimensions given by Planck's constant h. On the contrary, all the quantities named are always known inaccurately. If we increase the accuracy in one direction, the inaccuracy in other respects is increased. Thus in general both are only known inaccurately. This is the reason why the wave picture and the assumed existence of corpuscles can neither of them be completely carried through.

This "uncertainty principle"* was put forward by Heisen-

* Also called the "principle of indeterminacy".

berg in 1927, and was recognised to be the foundation of the quantum theory. Heisenberg at the same time suggested a number of experiments, all of which show that it is the duplex nature of light and matter, an experimentally proven fact, that sets this limit to our knowledge of the atomic world. We will now consider one of these experiments, namely, the making of observations by means of a "gamma ray microscope".

Say we wish to determine the position of a particle. This is easiest done by our being able to see it. In order to do this, we must illuminate it, since we cannot see it in the dark. In the case of ordinary bodies, which are large compared with the wave-length of light, we of course meet with no difficulties. The case is otherwise with small bodies of about the same size as the wave-length—smaller ones cannot be seen at all. We meet with diffraction phenomena similar to those known to us in connection with gratings. A very small body looks, by the light diffracted by it, like a tiny disc. We have already spoken of the little diffraction discs produced by the dust particles in the air when a ray of sunlight passes through a room. A cone of light proceeds sideways from the small body. If I regard this light as consisting of corpuscles, each ray of the cone is a possible track of a photon. The position of the small body is then known with an accuracy of the same order as the wave-length of the light used. Hence if we wish to determine its position very accurately, we must use light of very short wave-length. In the case of atoms, for example, we must use gamma rays. The fact that we cannot perceive these rays directly with our eyes through a microscope does not matter. We can easily perceive them by their action on a photographic plate. The practical difficulties which meet us in attempting such a measurement have nothing whatever to do with the argument, which is directed towards determining the inaccuracy in the determination of position remaining even when we leave out of account all difficulties due to our human inadequacy. Our conclusion is that such a determination of position can be made as accurately as the wave-length of the light used. The error a in the measurement of position is thus of the same order as λ

$$a \sim \lambda.$$

But in the case of a small corpuscle, we must take into account a factor which plays no part in the case of larger bodies. For light is itself corpuscular in structure, and consists of photons. A large body, as we know from the Compton experiment, is no more affected when struck by a light quantum than, let us say, the earth when struck by a football. But if the particle is an electron, it recoils; with this recoil, the electron acquires part of the impulse $p=\frac{h}{\lambda}$ of the impinging light; the recoil is greater, the smaller λ. Now since the diffracted photon may have travelled along any of the rays forming the small cone of light which enters the microscope, we cannot tell from the diffraction disc seen therein which direction the photon has actually taken. Hence, we are also unable to tell what fraction of the photon's impulse was given up to the electron, for this depends upon the direction in which the photon afterwards moves through the microscope. The inaccuracy in the impulse A is thus approximately of the order $\frac{h}{\lambda}$

$$A \sim \frac{h}{\lambda}.$$

The shorter the wave-length of the light, the more accurate is the determination of the position of the electron, and the greater on the other hand is the uncertainty regarding its impulse and its velocity. The product of the two uncertainties is

$$aA \sim \lambda \frac{h}{\lambda}.$$

$$aA \sim \lambda.$$

This is again the Heisenberg uncertainty relation.

This relation, it is true, plays no part in the world as directly perceived by our senses. Let us imagine a little ball the size of a pin head rolling along, its mass being one milligram ($m=1\times 10^{-3}$ gm.). Say we wish to know its position with an accuracy such that the error a is not greater than 1/1000 cm. ($a=10^{-3}$ cm.). We can only do so if we are willing to put up with an uncertainty in our knowledge of the impulse ($p=mv$), that is to say of the velocity v with which

the ball is moving. But this uncertainty is immeasurably small. For if we call this unavoidable error in the determination of the speed V, then

$$amV \sim h$$
$$10^{-3} \times 10^{-3}\ V \sim 6{\cdot}55 \times 10^{-27}$$
$$V \sim 6{\cdot}55 \times 10^{-21}\ \text{cm./sec.}$$

The velocity can thus be determined, in spite of the accuracy with which we are measuring the position, with a much smaller error than one-trillionth of a centimetre per second; in other words, with practically perfect accuracy.

If we are to see the track of an electron of mass 9×10^{-28} gm. in the atom (radius about 10^{-8} cm.), we need to use light of shorter wave-length than 10^{-8} cm., say X-rays. But if we cause an X-ray to fall upon the electron, in order that it may show us its position, it produces a Compton collision effect on the electron, and knocks it completely out of its track, indeed, right out of the atom. The electron is then no longer travelling in the track in which we wished to inspect it.

The Heisenberg uncertainty relation presents us with a new interpretation of the most important physical constant of atomic physics, the elementary quantum of action h. This is now seen to be a measure of the ultimate accuracy with which measurements in space and time, of energy, impulse and velocity, can be made; it sets us a limit in principle to the mutually consistent application of these concepts. Physicists had hitherto been innocent enough to believe that these concepts were valid everywhere in the world without restriction. They were imbued with a preconceived idea that the concepts of ordinary physics must hold without limit, even for the atom.

Physicists have now become wiser. If we are to understand the new results gained by experiments on micro-phenomena by applying to them concepts derived from several centuries of experimenting with macroscopic bodies, we cannot do so without some sacrifice. There is a limit in principle to the extent to which these concepts can be applied without self-contradiction. This limit is given by the Heisenberg uncertainty relation. This tells us that measurements made on

single electrons are fundamentally and inevitably inaccurate. Hence we can only make statements of a statistical nature concerning them. An exact statement concerning their behaviour as individuals is impossible, and we can only use statistics based upon the wave conception. The uncertainty relations depend, as we have seen, upon the dual nature of light and matter. They have nothing to do with the inadequacy of our present apparatus, for perfect apparatus would still not enable us to pass beyond these set limits. The means of measurement, the light in the case of the gamma ray microscope, influences the electron, the position and velocity which is to be measured, by striking it, and this influence can itself only be determined inaccurately since the track of the diffracted photon cannot be determined exactly. We are confronted with a somewhat analogous situation when we attempt to investigate our own mental processes.

If an angry philosopher were to try to use this state of his feelings to study anger in general, he would be unable to carry out his project. By turning his attention to the state of his feelings, he at once influences them to an incalculable degree.

De Broglie made use of a very appropriate picture to characterise our new enlightenment. If we lay together two glass plates each having a figure engraved upon it, we can see both figures at once with a not too powerful microscope. For if we focus this microscope on a point half way between the two figures, we can see both of them fairly sharply. But a high-power microscope will only give us a sharp image of one at a time. The more sharply we focus it upon one, the more blurred does the other become. "Classical mechanics corresponds to the low-power instrument, and gives us the impression that we can determine exactly both the position and velocity of the particle. But the new mechanics, which corresponds to the high power instrument, tells us that the localisation in space and time, and the description in terms of energy, are two different planes of reality, which cannot be both seen sharply at the same time."

CHAPTER VIII

Summary and New Discoveries

But all the story of the night told over,
And all their minds transfigured so together
More witnesseth than fancy's images,
And grows to something of great constancy;
But, howsoever, strange and admirable.
—HIPPOLYTA in A Midsummer Night's Dream.

We may now attempt to draw in broad lines a picture of the whole of physics, a picture no longer showing atomic processes as incomprehensible peculiarities, but comprehending in one unity both macro- and micro-physics. The concepts familiar to us in macro-physics cannot, as we have seen, be applied to atomic phenomena. We can do without them altogether, and use matrix mechanics. But we can also make use of them, and speak of bodies communicating energy and impulse one to another, and of waves spreading out with time into space. But we now know within what limits these two notions may be employed. Radiation and matter behave like bodies, inasmuch as they can only exchange energy and impulse according to the laws of bodies, and yet they behave like waves, since they spread out in space and time as if they were waves. The height of the wave crest at a certain point measures the probability of finding particles (photons or material corpuscles) at that point. These particles cannot be localised individually, and cannot in general be followed on their journey. The course of an individual is not determined; only statistical statements can be made about it. The uncertainty relationships give us, with the aid of the elementary quantum of action, a measure of the accuracy with which classical concepts may be used. The equations

$$E = h\nu \qquad p = \frac{h}{\lambda}$$

offer the possibility of at any time calculating from one set of quantities to the other.

Let us consider a few special cases. Firstly that of free motion without the action of any force. We imagine a swarm of electrons left to themselves without any influence from outside. Let us first assume that they all have the same exactly determined values of energy and impulse; then our equations tell us that λ and ν are also exactly determined; The swarm of electrons can be represented by a single wave. According to the uncertainty relationship, since we have supposed the error in the energy and impulse measurements to be vanishingly small, the distribution in space and time is completely undetermined. The wave is equally distributed over the whole of space. Particles may be met with anywhere.

If we now have instead of one wave several waves overlapping, we get a wave packet (p. 159); this is also called, in order to point out the significance of the wave, a "probability packet". If only a few waves of different wavelengths are superimposed, and hence the differences between λ and ν and so between p and E are not very great, the positions and corresponding times of the particles are still very indefinite according to the uncertainty relations. But the more waves of different length we cause to overlap and the less accurately therefore the impulse and energy are known, the more closely does the resultant wave draw together to a packet of small extent, and the nearer do we come to being able to speak of it as a corpuscle present at a definite point at a definite time. As time passes, the wave crest formed of the superimposed waves moves in space, and we have a particle in motion.

When the motion on the body takes place in dimensions so great that the quantum of action can be regarded as negligibly small, when, that is to say, the product of the errors according to the uncertainty relations has the value 0, it is possible to measure at the same time energy and impulse with practically absolute accuracy, and also to determine the exact position of the body in space and time. We are then dealing with what classical physics calls a "material point". The wave

packet representing it holds well together, as can be shown, on account of its large mass. This means that the probability of finding it anywhere else than upon the course so determined is vanishingly small; the material point moves along a definite track. We are also able to follow its motion along this track. The light used to illuminate it exerts upon it no perceptible force on account of its great mass.

Let us now consider what happens when forces of any kind act upon our swarm of particles. These forces will change the impulse p of the particles, and hence, on account of the equation $\lambda = \frac{h}{p}$, the wave-length of the corresponding matter wave, to an extent determined by the way in which the magnitude of the forces changes at different points of space. With change of wave-length, the velocity u of the waves, which is calculated from the equation $u = \lambda\nu$, will be different at different points. The wave no longer moves forward with an unchanged front as in the case when forces were absent, as a "plane" wave, but is curved. If the field of force does not change too rapidly from point to point so that the change in it is only perceptible after many wave-lengths, the wave-lengths may be regarded as small compared with the dimensions of the field of force. We then have the same case with our matter waves, as with light waves. In regions large compared with the wave-length of light, the latter may be regarded as rays of small particles. In a corresponding way, wave mechanics passes over in this case to classical mechanics. We can speak of bodies moving in paths possessing a curvature caused by the forces present. If forces are absent, we have again the case already considered. The bodies move in straight lines with uniform velocity.

A swarm of cathode ray particles flying through a magnetic field, or of alpha ray particles shot into atoms, may be successfully treated, as was already done by classical physics, as a number of macroscopic bodies moving according to the laws of ordinary mechanics. The wave-lengths of the particles are in these cases small compared with the dimensions within which the magnetic and electric fields, through which they are passing, change.

But the picture is altogether altered when we consider the motion of electrons in the interior of an atom. The wavelength of the electrons is easily calculated to be of similar size to the radius of an atom in its normal condition, and at this distance, the attractive force of the nucleus changes very rapidly. Here therefore we require to use strict wave mechanics, which now shows its very great superiority to classical mechanics. For it gives the same results as the latter in dealing with large bodies, but it is also able to master the phenomena of atomic physics. We have seen this to be true in the case of the hydrogen atom. We know that wave mechanics teaches us to regard a hydrogen atom as a system of stationary waves, which can only be formed, as we learned in Chap VI, with quite definite values for the energy. These probability waves flow around the nucleus of the atom. Concepts such as place, velocity, path of electrons have no longer any validity. If we look for actual electrons in the interior of atoms, we meet, as someone jokingly remarked, with the fate of Faust, when he visited the witches.

The excellent quantitative agreement with experience of the wave mechanics of the hydrogen atom has already been discussed. The greater the measure of truth contained in it, as compared with former theories, is shown by the fact that it can deal successfully with problems which had hitherto resisted all attempts at solution. We will now consider some of the most important of these.

These applications of wave mechanics are matters which can really only be understood mathematically. For the mathematics no longer serves the sole purpose, as in classical physics and the early quantum theory, of giving us numerically correct results concerning processes which can be grasped logically without mathematics, and pictured concretely in a qualitative way. Since the concrete concepts of the macroscopic world are valid in the micro-world only within the limits set by the uncertainty principle, the mathematical conclusions of quantum physics contain everything that can be relevantly said concerning the processes. We will nevertheless attempt to state the novel, and hence not very simple, ideas contained in these mathematical deductions,

naturally without attempting to give the calculations themselves.*

The Bohr theory could only be used for accurate calculation in the case of the hydrogen atom. The spectrum of the next most simple element, helium, an atom with two charges to its nucleus and two electrons, could only be explained qualitatively. Its most important properties were found to be the following; the element exists in two different states, the para-state with simple, "singlet" energy levels, and the ortho-state with three "triplet" levels close together. A one-quantum fundamental state exists only for parhelium. The transformation of ortho- and parhelium into one another with emission of light, only occurs very rarely. Bohr's theory did not allow us to pass beyond this qualitatively correct analysis of the character of the helium spectrum to correct quantitative deductions concerning it.

What have the new ideas of wave mechanics to tell us about the helium spectrum? Heisenberg has dealt with this problem. We will first attempt to understand whether and how the spectral properties of helium can now be deduced. The helium atom presents us with the well known "three body" problem which also occurs in the calculation of the orbits of the planets around the sun. In this case, the nucleus and the two electrons all exert mutual forces. We attempt, like the astronomers, to approach the truth step by step. The interaction of the two electrons could only be small, and can later on be taken into account by a perturbation calculation.

We may thus hope to obtain the main result by considering the two electrons as influenced, not by one another, but only by the nucleus. We then have a case similar to that of hydrogen, excepting that the nucleus now has a double charge. Bohr's theory required us to work out a time table for the electrons, in order to avoid a possible collision between them, since their paths cross. But no grounds for any special assumptions of this kind were given by experimental experience. Wave mechanics needs no such assumptions; the electron distribution is now given by two independent waves,

* Short elementary works: H. T. Flint, *Wave Mechanics* (1931); R. W. Gurney, *Elementary Quantum Mechanics* (1934).

each in three dimensions, the two together being in a six-dimensional space. No concrete picture can of course be formed of this arrangement. But one can draw good logical conclusions even in six-dimensional space, and therefore also perform calculations. Each of the two waves is, as in the case of hydrogen, determined by three quantum numbers, and has a very different shape according to the magnitudes of these numbers. In the Bohr model, we should have now to combine the distribution of the electrons on different orbits as given by the values of the quantum numbers. In the new model, combined vibrations in six-dimensional space are formed. To each possible state of vibration there belongs a definite value for the energy. We now come to an important point. If we interchange the two electrons, which are in different states of vibration, the waves in the configuration space are differently arranged, but, since two electrons are exactly similar, the energy of the atom must not be influenced by this interchange. There are therefore two different states of vibration having the same energy.

That is also true when several such waves of the two distributions overlap. The wave crests add together, or when the waves are vibrating oppositely, they subtract. It seems as if an almost limitless number of waves should be possible. However, a limit is set by the interchangeability of the electron. Let us consider this a little more closely. The new mechanics is essentially statistical in nature. We are always dealing with a large number of atoms, whose probable electron distribution is given by the wave crests, or equally large wave troughs, of the matter wave. Also, the two electrons belonging to a certain atom are exactly alike and not distinguishable. Hence only such combinations of waves are possible as comply with this condition. If x denotes the height of the wave crest and hence also the depth of the wave trough of one of the two overlapping waves, and y denotes the same for the other, we cannot for example have a wave formed by the overlapping of two similar waves of the first and three of the second kind, and hence possessing crests and troughs of a size $2x+3y$. For the combined wave resulting from interchange of the two electrons would then have a

different crest of height $2y+3x$. The only possibilities for the combination of the two electron waves are given by the expressions

$$x+y \text{ and } x-y.$$

In the first, "symmetrical", case an interchange of the two electrons produces no effect, for we get the same wave $y+x$. In the second, "antisymmetrical", case, we get the expression $y-x$, which is opposite to $x-y$; the height of the wave crests and the depth of the troughs, which give the probability of the electrons' position, is the same in the two cases. Only such waves oscillating in six dimensions can be formed, as are either symmetrical, that is are unchanged by interchange of the two electrons, or antisymmetrical, and are transformed into the opposite value. Both states are characterised by the same energy when the frequency is the same.

The two states of vibration of the same energy are slightly modified when the interaction of the two electrons is taken into account, that is, when a small disturbing additional field of force is introduced. This phenomenon is known as quantum mechanical resonance. The mutual influence can be calculated, and it is found to be slightly different as between the symmetrical and antisymmetrical cases. The energy levels are slightly lower in the second case than in the first. Hence wave mechanics also leads to two different states for helium. According to our earlier description, the antisymmetrical, that is the poorer in energy, must belong to orthohelium, and the symmetrical therefore to parhelium. In the most stable state of least energy, all quantum numbers must be alike for both electrons, and have the lowest possible values. But the quantum number determines the fundamental frequencies. These must therefore be equal, for $x=y$. This fundamental state in which the principal quantum number $n=1$, can therefore only exist when the wave function is symmetrical. It is then $x+x=2x$, that is to say, for parhelium. Orthohelium has no one-quantum state, for in its case we have $x-x=0$.

In the formal atomic model, the one-quantum orthohelium state was "excluded" by the Pauli principle. This asserted that two electrons of an atom could never agree in all four

quantum numbers. We know to how great an extent this principle gave us correct results; for example, only by its aid did we begin to understand the periodic system of the elements. The only unsatisfactory feature was that the principle itself could not be proved; that it was introduced as a foreign body into the quantum theory. There is no change in this respect when we come to wave mechanics. We are still unable to give the Pauli principle a more fundamental foundation; we can only formulate it in the new terms, and apply it as an additional postulate. How is the Pauli principle to be stated in the language of wave mechanics? Since any two electrons of an atom must differ by at least one quantum number, the states of vibration must be changed by interchange of these electrons. But a state of vibration symmetrical for the two electrons would not be altered by their interchange, and hence such a state cannot exist. The Pauli principle therefore states that only such states of vibration are possible as are antisymmetric for all electrons. According to this, all the parhelium states would be impossible.

Now we know that this is not the case. But there is a further refinement still to be introduced. The helium spectrum has also a fine structure, which has to be explained. Its lines and hence its energy levels are in part multiplex. The para-levels are simple (singlets). The ortho-levels exhibit three lines close together; they are triplets. In the first quantum theory, this fact was explained by the additional assumption of electron spin, the rotation of the electron about its own axis. But if the corpuscular conception of electrons, with their orbits and their jumps from one to another, is to be excluded from wave mechanics, what right have we to ascribe to these electrons, which have now been resolved into a system of waves, a rotation about their own axes? The quantum physics developed by Schrödinger and Heisenberg does not lead to any electron spin. But this difficulty no longer exists. Dirac has succeeded in devising more general wave equations which conform with the theory of relativity. In these equations, without the necessity for making any special hypothesis, an additional magnetic energy appears such as is the property of a rotating electrified body. If therefore we speak of cor-

puscles, we must regard the magnetic field as arising from electrons rotating about their axes. This is the wave mechanical description of electron spin.* So, without forming a new hypothesis, we may thus regard a small additional magnetic field as accompanying the electrons. This field exercises a further weak influence on the height of the energy levels. The probability of the electron distribution was given by our electron wave. We now must take into account a further probability, namely that governing the direction of rotation of the two electrons of the helium atom, whether similar or opposite. This probability is also given by a wave. It depends upon the spin quantum number, which may have either of the values $+\frac{1}{2}$ or $-\frac{1}{2}$. Let x represent the height of the probability wave when the two electrons have spin $+\frac{1}{2}$, y the height when they have spin $-\frac{1}{2}$. Let u and v be the heights of the wave, when one electron has spin $+\frac{1}{2}$ and the other $-\frac{1}{2}$, and conversely. Since the two electrons cannot be distinguished from one another, we have as possible waves in the two latter cases again only $u+v$ and $u-v$, just as in the previous case of the waves of probability of position. There are therefore three symmetrical spin waves x, y, $u+v$ and one antisymmetrical, $u-v$.

The waves determining the probability of position combine together with the new waves giving the probability of distribution of the spin. When I throw a die, the probability that I throw a three is 1/6, for this case is one of six possible cases; the probability that I throw a four with a second die, is also 1/6. If I throw both together, and one gives a three and the other a four, we have a case representing 1/6 of 1/6 of all possible throws. The combined probability is equal to the product of the two single probabilities, $1/6 \times 1/6 = 1/36$. In the same way, the probabilities of the electron distribution and the electron spin are to be multiplied together. According to Pauli, the whole wave must be antisymmetrical; this is only the case when one factor is symmetrical, and the other antisymmetrical. Hence, as opposed to the conclusions which we drew before considering the spin, para-states must be possible

* The Dirac theory also leads to a complete explanation of the anomalous Zeeman effect, a fact that we can only mention here.

as well as ortho-states. In them, the symmetrical wave of position combines with the antisymmetrical wave of spin. In parhelium the electrons have opposed spins. An ortho-state on the other hand is obtained by a combination of an antisymmetrical position wave with one of the three symmetrical spin waves: while the para-states were all singlets, the energy levels of orthohelium are in sets of three close together, triplets in other words. In these states, the electrons have like spins.

These mathematical considerations have naturally been somewhat long-winded, since we have been avoiding mathematical symbols as far as possible. We get the same result as with the former theory. The fact that quite different theories are seen to explain the same set of facts to a very large extent is very instructive as regards the real meaning of physical theory. The earlier theory would obviously be preferable if we judge by the ease with which a mental picture can be formed from it. It is not so easy to become accustomed to these probability waves overlapping in six dimensions. But if we now proceeded to carry out the extremely tedious calculation of the energy values, as Heisenberg and others have done, we should find that wave mechanics, as opposed to the earlier theory, leads to correct results. The problem of finding by calculation correct values for the states which helium can assume defeated the first quantum theory, but is solved by wave mechanics. This fact is decisive in favour of its greater value.

As regards the higher atoms following helium, the mathematical principles have already been thought out, but the calculations become more and more difficult, and have not yet been performed.

A further problem of the first importance has been successfully attacked by wave mechanics: the old problem of the nature of chemical forces. We have learned that Kossel already succeeded in using the Bohr theory to explain chemical forces on an electrical basis in many cases, as forces acting between oppositely charged atoms. But the riddle remains that atoms of the same kind, such as several carbon atoms, are able to combine together. Nevertheless, it is just these

combinations which occur in enormous variety in the substances composing the bodies of all living creatures. We cannot conceive what forces atoms not possessing opposite electrical charges can exert upon one another. When neutral, they should surely be unable to exert any external action. The attempt was made to explain the simplest case of this kind, the combination of two hydrogen atoms, along other lines. Arrangements were suggested of the two electrons and the two nuclei which would result in equilibrium under the mutual repulsive and attractive forces of the four bodies. It should then be possible to calculate the energy levels given by the spectrum of the hydrogen molecule from the various possible arrangements. But the uncertainty principle tells us that this procedure is illegitimate. The position of the electrons on certain orbits, and also their velocities, are specified exactly, while at the same time the attempt is made to state exactly the energy of this micro-physical structure. If we wish to use quantum physics to calculate the possible energy states of the molecule, we must renounce any determination of the electron in space and time, and fall back upon statistical statements concerning the probable distribution of the corpuscles.

Heitler and London attempted in 1927 to discover the energy levels of the hydrogen molecule by this means. This simplest case should decide whether wave mechanics is able to answer in principle the question concerning the nature of chemical combination between neutral atoms; also the nature of these forces should be brought to light.

The two investigators first imagine the atoms composing the hydrogen molecule to be so far apart as to exert no sensible effect upon one another. The total energy is then just twice as great as the known energy of a single hydrogen atom. If the two atoms are in different states, an exchange of the two electrons will alter exactly as in the case of helium the wave describing the complete system without the total energy being thereby changed. Here also, the overlapping of such waves can only result in symmetrical or antisymmetrical waves. They have the same frequency, and hence the same energy.

If now the two atoms approach one another, the mutual disturbing effect will become greater as the distance apart

becomes less. The energy belonging to the symmetrical wave is differently influenced from that belonging to the antisymmetrical wave. Heitler and London were able to calculate the variation in the two energies with variation in distance apart of the nuclei. Fig. 53 shows us that in the antisymmetrical case, an approach of the two atoms can only be brought about by continually increasing expenditure of energy: in the symmetrical, on the other hand, they approach one another with loss of energy until they are within a certain distance, which by calculation is $0{\cdot}8 \times 10^{-8}$ cm. In order to bring them closer, it is then necessary to supply energy. The case is like that of a ball which has fallen into a cup; it remains there until it is given an impulse from outside. Hydrogen atoms in the symmetrical wave state approach one another of themselves, that is to say without the supply of external energy, until they are separated only by the distance given above, and then remain in this position unless disturbed from outside.

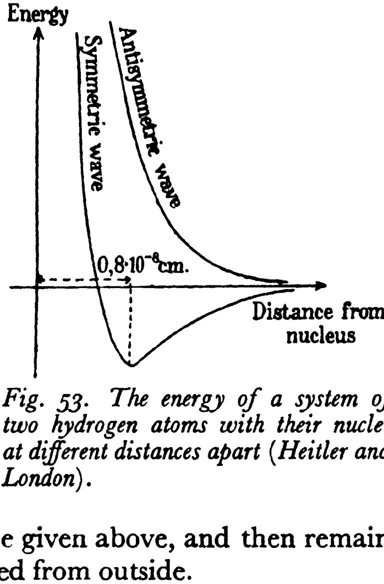


Fig. 53. The energy of a system of two hydrogen atoms with their nuclei at different distances apart (Heitler and London).

Chemical combination of two atoms to form a molecule has taken place. The energy required to separate this molecule into its parts has been calculated by various methods of approximation, and the results agree satisfactorily with experiment.

Only one difficulty remains. According to Pauli's principle the symmetrical electron wave ought not to exist at all. As in the case of helium, we can only solve the difficulty by taking into account the spin of the electron. The symmetrical or antisymmetrical wave so far calculated, which measures the probability of position of the two electrons must be multiplied with another wave measuring the spin state, and likewise

either symmetrical or antisymmetrical. Then only two cases are possible: antisymmetrical position waves and symmetrical spin waves unite to give repulsion of the two atoms, while chemical combination occurs when the position wave is

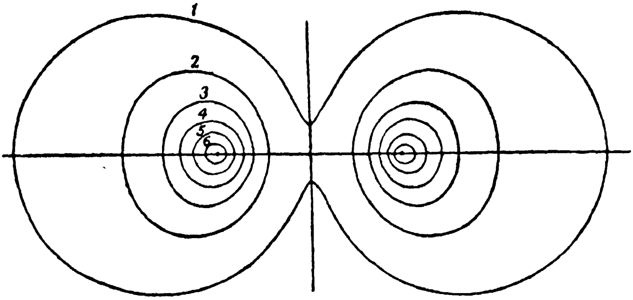


Fig. 54 a.

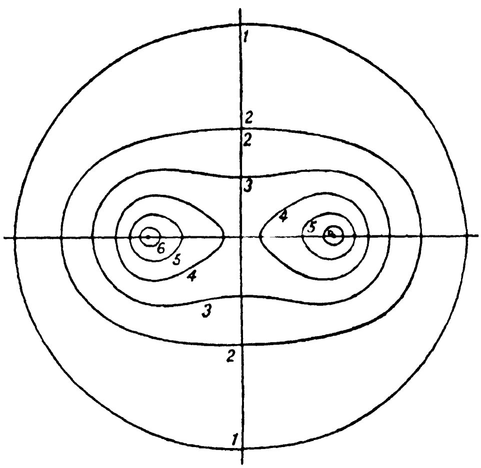


Fig. 54 b.

Fig. 54. *Probability waves of hydrogen:* (a) *antisymmetrical,* (b) *symmetrical* (*London*).

symmetrical and the direction of the spins of the electrons is opposite. Figs. 54 *a* and *b* show the curves of equal density of the probable position of an electron in the antisymmetrical and symmetrical cases.

The cause of the chemical combination of two hydrogen atoms is thus not to be sought in electrical or other forces

known to classical physics. Nothing throughout the latter suggests an explanation of this combination. It is a quantum phenomenon which cannot be understood by means of the concepts and ideas of the old physics.

London and others, starting from this point, have also attempted to explain the other chemical combinations between like atoms. The formation of a molecule always occurs in the case of hydrogen when two electrons of two atoms which meet have opposite spins. Without going more deeply into chemistry, we cannot discuss this question further. But this example of the hydrogen molecule is enough to teach us what "understanding" a process means. It means finding a place for it in a comprehensive set of ideas, which enable it to be calculated beforehand. But the place need not be one in some particular world of ideas with which we have long been familiar. Newton did not explain the phenomena of planetary motion by finding a place for them in the old familiar ideas concerning collision and pressure, but by bringing them into his own new set of concepts, which he mastered by means of mathematical methods invented by himself. The phenomenon of chemical combination between like atoms is "understood" when a place is found for it in wave mechanics.

Molecules, including hydrogen molecules, emit spectra different from, and much more complicated than, those of their atoms, when they are caused to become luminous, as physicists have long known. The first quantum theory explains the numerous lines of these "band spectra", as due to energy changes in new quantised motions which are now added to the switches of the electron between permitted orbits. These quantised motions comprise oscillations of the nuclei relatively to one another, and rotation of the line joining the two nuclei about an axis at right angles to itself. These quantised oscillatory and rotational movements of the concretely imagined nuclei are replaced in wave mechanics by probability waves, exactly as was done with the motion of the electrons in atoms. These waves are obtained when we take account of the interaction, hitherto neglected, between the nuclei and electrons of the hydrogen molecule. We then

get the same expressions to describe the band spectrum as were given by the earlier theory. The new formulae also exhibit a few small improvements, which are justified by experiment, but cannot be deduced from the earlier theory.

The spectral lines further show plainly that the total probability wave can be both symmetrical and antisymmetrical. Once more, Pauli's principle would exclude the symmetrical wave. Hence, we must again look for a further small disturbance corresponding to the electron spin in the case of helium. We therefore now ascribe spin to the nucleus, and this leads to three symmetrical and one antisymmetrical wave. The probability of a given nuclear spin then multiplies again with the probability giving the configuration of the molecule, so that according to Pauli only antisymmetrical waves are possible. Hence two kinds of hydrogen must exist, one having opposite nuclear spins, and called parahydrogen, and the other, occurring by the way three times more frequently, having similar nuclear spins; this is called ortho-hydrogen. Ordinary hydrogen is a mixture of the two kinds.

This phenomenon, which is quite unexpected on classical ideas but is predicted by wave mechanics, has been confirmed by experiment. In the first place, the specific heat of the two kinds of hydrogen can be calculated—the heat, that is to say, that must be given to a gram of the substance in order to raise its temperature by one degree. This specific heat is found to be different for the two kinds of hydrogen molecules, and also to vary differently with temperature. The calculated variation of specific heat with temperature of a mixture of 25 per cent. parahydrogen and 75 per cent. orthohydrogen, is exactly that found for ordinary hydrogen.

But there is more to come. The proportions of the mixture, 1 : 3, are very difficult to influence, as the theory also predicts. But after the lapse of a considerable time, the percentage of parahydrogen would become greater and greater the lower the temperature; at 21° absolute, that is to say at −252° C. below freezing point, all hydrogen molecules should change, given time, into the para form. This transformation has been successfully accomplished. Eucken brought it about

in 1929 by using high pressure. Bonhoeffer and Harteck at the same time effected it by saturating carbon with liquid hydrogen and then allowing the latter to evaporate. They were then able to show that the peculiar properties of pure parahydrogen agree with the theory, for example the change of its specific heat with temperature, its thermal conductivity, and other properties.

This entirely unexpected prediction of the existence and properties of two kinds of hydrogen is a great success of the modern quantum theory. Another subject hitherto almost completely closed to us has been entered by quantum physics. This is the physics of atomic nuclei. We learned in the first chapter that the nuclei of the heaviest elements fall apart of themselves, and furthermore, in a way in which we are unable to influence. We can exert no effect of any kind, not even upon the rate of decay, which is extraordinarily different for different radioactive elements. Some elements decay very slowly indeed. In the case of uranium, as we have said, almost 5 milliards of years are required for half of the original atoms to be transformed, while in the case of radium A this occurs in about three minutes, and of radium C in about one hundred-millionth of a second. It has further been found experimentally that as these "half-periods" decrease the energy of the alpha particles emitted increases, though to a very much smaller extent. This extent was also known, but no reason for radioactive decay, let alone for the actual rates, could be given.

For classical physics, this decay was quite incomprehensible. When we bombard the nuclei of the heavy atoms with positively charged alpha particles, the nucleus, which is likewise positively charged, repels the alpha particles. This repulsion increases up to a minute distance of three-billionths of a centimetre, within which distance the fastest alpha particles approach the nucleus; this is readily understood on macrophysical lines. Perhaps the law of repulsion also holds for even smaller distances, but at some point at a very small distance indeed from the centre of the nucleus, something else must happen. The repulsive force must be decreased, and be changed on closer approach into attraction. For the alpha

particles contained in the nuclei are not repelled, but on the contrary held so firmly that most nuclei do not allow them to escape at all. Fig. 55 shows how the energy with which an approaching particle is repelled increases as it approaches the nucleus.

Hence we must conclude from what has been said that the curve must fall steeply very close to the nucleus, though we do not know its exact form. Within the nuclear distance 3×10^{-12} cm., there must be a steep peak. If this peak is higher than the energy possessed by an alpha particle contained in the nucleus, it forms a barrier over which a particle

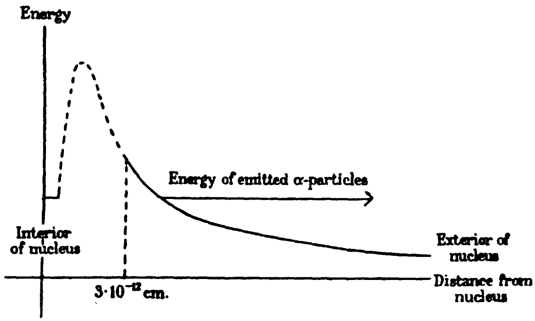


Fig. 55. The curve, the full line part of which is determined by experiment, shows the energy possessed by an alpha particle at different distances from the centre of the nucleus (Gamov).

cannot climb, according to classical ideas, and hence must remain permanently a prisoner. Outside and inside it can exist, but it cannot climb the wall, which for this particle is too high. If it is inside, it can never escape into the freedom of the outer world. The stability of atomic nuclei can be understood on classical lines, but not the fact that alpha particles possessing an energy smaller than the height of the barrier, can occasionally break through it. The energy of these particles is easily determined by measuring the speed at which they are emitted.

But quantum physics is also able to tell us why a particle is often met with outside the barrier. Gamov, and, independently of him, Gurney and Condon, worked out the theory. According to classical physics the position of an

alpha particle is found by following its track from the interior outwards. But wave mechanics will have nothing to do with the idea that particles possessing paths which can be followed exist in the micro-world. The probability of finding alpha particles inside or outside the atom is given by a wave. This wave extends to infinity, although its height decreases very rapidly. It cannot be confined by barriers, but penetrates them, although its height rapidly decreases. Hence there is a certain though very small probability that an alpha particle may be found outside the highest barrier. And we should be able to calculate correctly from the energy possessed by the alpha particle when found outside, the degree of probability of its escape from the nucleus, that is to say, the half-period of the atom. This calculation has been actually found to agree with the very various values of known half-periods, as regards order of magnitude and also in its relation to the energy of the alpha particles.

But if particles of less energy are able to get out of the interior of atomic nuclei, in spite of the high wall by which they are surrounded, even if only in small quantities, we must conclude that the opposite case is also possible; it should be possible to force particles of low energy to enter the nucleus from outside. The old problem of the alchemist, the transformation of one kind of atom into another, thus gains new possibilities of solution. We described at the end of the first chapter Rutherford's experiment in shattering atomic nuclei by means of alpha particles of very high energy. According to wave mechanics, it should also be possible to fire into the fortress with much weaker artillery, and to cause disturbances to take place inside. For even a weaker projectile, say the positively charged hydrogen or helium atom, which we get in a discharge tube, has also a wavelike character. Its wave tells us the probability of finding it in any given position. Since this wave permeates, although with diminishing intensity, the whole of space, there is a possibility, very small, it is true, that a projectile of this low energy may reach the interior of the nucleus.

Charged helium atoms, if they are to be given the energy of the alpha particles used by Rutherford to fire into the

atom, would require to be discharged through an electrical voltage of several millions of volts. The German physicists Brasch and Lange were the first to attempt to use the high voltages occurring during thunderstorms, and they built an apparatus for this purpose on Monte Generoso, on the shores of Lake Lugano. Their plan was to charge condensers to a very high voltage, and then allow them to discharge suddenly through special discharge tubes, and so supply the necessary energy to the hydrogen nuclei. But these high voltages are very difficult to manage experimentally. According to wave mechanics, as we have seen, a small fraction of particles of considerably less energy should also succeed in reaching the interior of atomic nuclei.

But when protons, even those of less energy, have once forced their way into the nucleus, a much more powerful effect is to be expected, for they carry with them a secret store of energy. This is due to the fact, at first sight very strange, that a proton loses mass when it becomes part of a nucleus. A proton inside a nucleus is lighter than one outside. The latter weighs 1·0072, as compared with the oxygen atomic weight 16·000. The weight of the inside proton can be calculated by comparing the masses of two nuclei which differ in their structure by exactly one proton, for instance, carbon (C = 12·0036) and boron (B = 11·0110). The difference of these weights is 0·9926, and this is the weight of an inside or nuclear proton. This is thus 0·0146 smaller than the weight of a proton free outside. This mass defect (see p. 11) which occurs when a proton is built into the nucleus, corresponds to a very considerable amount of energy, as is easily calculated. This energy is much greater than that set free when an alpha particle gets into a nucleus. It is available when a proton is successfully fired into a nucleus.

This consequence predicted by wave mechanics has actually been realised. In the year 1932, Cockcroft and Walton were able to attack the nuclei of lithium, boron, and other elements successfully with protons, driven by voltages of only a few hundred thousand. The tracks of the disrupted atoms could be photographed, and measured by means of the cloud

chamber (p. 23). The transformation takes place according to the equations:

$$Li_3^7 + H_1^1 \rightarrow 2He_2^4$$

$$Bi_5^{11} + H_1^1 \rightarrow 3He_2^4.$$

The upper figures stand for the atomic weight, the lower for the nuclear charge. Other physicists have shown that voltages of 30,000 and even less are sufficient to fire protons successfully into nuclei. These nuclear transformations, in which protons are used to force alpha particles out of the atom, are the opposite numbers to Rutherford's first transformation (p. 25) in which alpha particles drove out protons.

We seem thus to have found in quantum physics the key not only to the nature of chemical combination, but also to an understanding of the processes going on in the nucleus, although we have hitherto only been able to open a few doors with it. These last examples once more show us plainly how limited is the applicability of the notion of a "body" in the micro-world, and also, that the fate of an individual body remains completely undetermined, and can only be stated in the statistical terms of a wave conception.

When we once more consider all atomic transformations hitherto made, we perceive that the number of atoms actually transformed is always very minute. This somewhat damps our delight that the dream of the alchemist has at last been realised in a number of cases. The probability that one of our projectiles, in spite of its concentrated energy but minute size, should, in passing through millions of atoms, actually hit a nucleus, is extremely small. An enormous quantity of ammunition must be expended in order to score a rare hit on a nucleus. The resulting fragments themselves have, it is true, very high energy. But there is not the slightest prospect that this energy will ever be of practical use in spite of the prophecies of people less familiar with the subject. Rutherford says that anyone who looks for a source of power in the transformation of the atoms is talking moonshine.

Too great attention in scientific work to practical advantage is always finally punished. Nature is not constructed in such a way as to make life easy for man. Man is a modest,

humble part of nature, and should be less alert to seek his own advantage. All the laborious investigations of which we have spoken have been carried out in a very different spirit, a spirit much more ideal and less selfish. Like all true scientific investigations, they were guided by the sole aim of discovering the truth; in this case, the truth about the forces which finally hold the world together. The time for making practical use of this research will come when it is due. Whether that will be to-day or to-morrow, or in a thousand years, no one can say. The research worker can do no more than work as everyone should, whatever his occupation, with single-minded devotion to the matter in hand.

Radioactivity is to-day in a state of rapid development. The investigation of the structure of the outer shell of electrons is being followed by the study of the nucleus. We may hope that the greatest period of radioactive investigation is only beginning. In 1906, the Rector of Freiburg University, Himstedt, gave an address on radioactivity and the constitution of matter, a copy of which was solemnly deposited in the foundation stone of the new University building for the benefit of posterity. This gesture was symbolic of our conviction that radioactivity will give us decisive information concerning matter, and we have indeed already learnt very much that is new. But physics in this department is still at the stage of discovering, collecting, and provisionally arranging facts. Theory is feeling its way with classical and quantum notions in these problems, but it is still unsure of itself, and always prepared to find that entirely new conceptions are suddenly required.

Important experimental discoveries have been made in the last few years. In the first place, Bothe and Becker found in 1920 that the nuclei of the lighter elements such as boron, lithium, and above all beryllium, when exposed to the alpha rays of the radioactive element polonium, emit an especially penetrating gamma radiation.

Irene Curie, the daughter of the discoverer of radium, and the French physicist Joliot thereupon showed that the radiation thus released from beryllium has quite a peculiar action which cannot be explained by the gamma radiation.

When it passes through hydrogen compounds such as paraffin, it occasionally ejects from these substances protons of high energy, and also sets light atomic nuclei which it meets in rapid motion. The tracks of these protons and other nuclei can be followed in the Wilson cloud chamber. Shortly afterwards, it was found that nuclei struck by the new radiation are sometimes disrupted, and then the tracks of both fragments can be followed (Fig. 56, opposite p. 198).

But the projectiles derived from the beryllium never leave tracks in the cloud chamber. This fact proves that these unknown projectiles cannot be electrically charged particles, for only such can form cloud tracks. Nevertheless, the active agent comes from the beryllium, for it can be shown that the tracks of the atomic fragments and the trajectory of the unknown projectiles all lie in one plane, as should be the case with any kind of collision of the particles. Chadwick found the explanation by determining the mass of the particles. He showed that the laws of conservation of energy and impulse which must hold for the collision would be violated if the gamma ray photons emitted by the beryllium are regarded as the active projectiles. It thus follows that the beryllium must be emitting something else besides gamma rays. The laws of collision lead to a mass 1 for these electrically uncharged particles of matter; in other words, the same mass as that of a proton. These particles of mass 1 and charge 0 were called by Chadwick *neutrons*, following a suggestion of Rutherford, who had already sought for such particles in the year 1920. This assumption of a new kind of atomic constituent is actually able to explain all the observed phenomena. Since the neutrons are without electric charge, they cannot, as we have seen, make cloud tracks; they fly unhindered, and undeviated through the electron shells of all atoms; only when they occasionally strike a nucleus full on do they drive it off as one billiard ball drives another, or on occasion shoot parts of it out. Such effects only occur very rarely; it has taken a generation of work at radioactivity to discover the neutron. This is readily understood when we consider that neutrons which have formed part of a beryllium nucleus cannot themselves be larger than this nucleus, and so must

Fig. 56. Disruption of an oxygen nucleus by means of a neutron in a cloud chamber. The electrically neutral neutron leaves no cloud track. The fine ray to the right is a helium nucleus shot out of the atom, while the short thick track to the left belongs to the remainder of the atom. (Photograph by Irene Curie and F. Joliot in Journal de Physique et le Radium, *January, 1933.)*

[*to face page 198*]

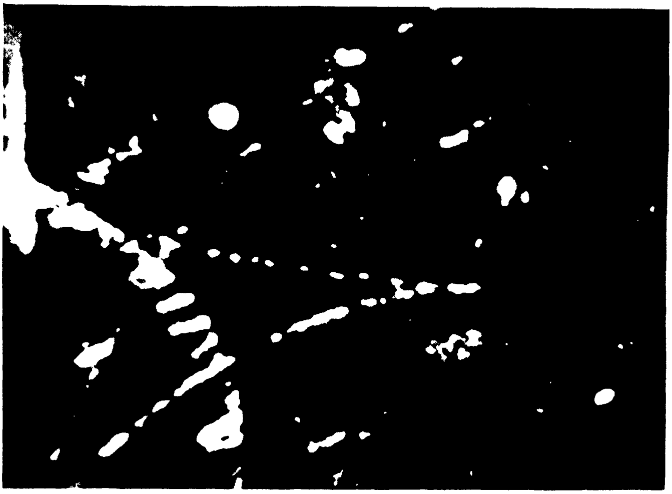


Fig. 57a. *A photon strikes an atomic nucleus in a cloud chamber, and is transformed into an "electron pair", electron + positron. (Photograph by Irene Curie and F. Joliot, in* Journal de Physique et le Radium, *August, 1933.)*

[*to face page 199*]

have a diameter of about 10^{-13} cm. It is for this reason that the only observable signs of life which it gives, namely the blows which it deals to other atoms, are so very rare. The production of the neutron (Nn) from beryllium by bombardment with alpha particles is described by Chadwick in the following equation:

$$\mathrm{Be}_4^9 + \mathrm{He}_2^4 \rightarrow \mathrm{C}_6^{12} + \mathrm{Nn}_0^1.$$

A neutron is exchanged for an alpha particle; in the disruption illustrated in Fig. 56, the converse takes place, and an alpha particle is replaced by a neutron.

These new constituents of the atom can also be shot, like alpha particles and protons, out of the nucleus of heavy hydrogen (p. 101), now called a "deuton", or "deuteron". These deuteron projectiles are obtained by applying an electric voltage to a tube filled with heavy hydrogen under reduced pressure. If they are then fired at other heavy hydrogen nuclei, a particularly rich yield of neutrons is obtained.

Shortly after the discovery of the neutron, a still more remarkable discovery of a further constituent of atomic nuclei was made, a discovery of something hitherto regarded as impossible. The American physicist Anderson was investigating cosmic radiation. He allowed this to pass through a cloud chamber, and observed the action of a magnetic field upon the cloud tracks which it produced. Among 1300 tracks, he found fifteen which were bent by the magnetic field in the opposite direction to electrons (Fig. 57, title page). They must therefore possess a positive charge. By allowing these rays to pass through thin metal plates he was able to estimate, from their loss of energy, that their charge was about unity, that is equal to that of a proton, but that their mass was very much smaller, and probably equal to that of an electron. Also, the tracks which they leave are loose and thin, and not thick and strong like those of protons and alpha particles. Anderson had discovered the positive atom of electricity in a state free from matter, in other words, the opposite number to the electron. He called these particles "positrons". We must suppose that they are released from

atomic nuclei by the action of the cosmic radiation. The same discovery was made simultaneously in England, by Blackett and Occhialini. Shortly afterwards, it was found that all hard gamma radiation causes matter, and particularly heavy atoms, to emit positrons. And quite recently they have been found to be emitted by radioactive material. A very important point in this discovery was the proof that positive electric charges can exist unassociated with matter. It is further of extraordinary interest that the English physicist, Dirac, had already predicted the existence of positrons four years before their discovery.

We have learned (p. 184) that Dirac succeeded in setting up relativistic wave equations. He was able by their means to explain the existence of electron spin, and hence the separation of spectral lines by magnetic fields. But a further consequence of this theory seemed at first to throw doubt upon its truth, for it leads to the statement that electrically charged elementary particles should be capable of passing over into states in which their energy is negative, as opposed to the ordinary and known states in which their energy is positive. Further, each of these very numerous states of negative energy should be assumed by a different elementary particle according to Pauli's principle. These states of negative energy required by Dirac's theory have naturally never been observed. They might nevertheless exist, for, as Dirac says, nature might be so constituted that a uniform occupation of all states of negative energy would be unobservable, but that an unoccupied state of negative energy, since it represents a departure from uniform occupation, should be observable. This unoccupied "hole" in the abundant states of negative energy would be a state of positive energy, for in it, *negative* energy would be *missing*. Now Dirac's theory leads to the result that in this case the elementary particle would have a positive charge. We should thus be able to observe positrons. It should therefore be possible, by using radiation of sufficient energy, to convert an electron from among those of negative energy into a state of positive energy, with the result that an ordinary electron would appear. At the same time, a hole would be produced in the states of negative

energy. From what has been said, we should then expect the hole to be observable in the form of a positron. It should thus be possible to produce matter by means of radiant energy. And such "electron twins" have actually been found experimentally, as we see from Fig. 57*a*. The transformation must take place according to the fundamental equation of the relativity theory, which connects the mass of matter m, the energy of radiation E, and the velocity of light, c, thus: $E = mc^2$.

We are thus able to calculate the amount of energy required from the gamma radiation to create a twin electron pair, electron and positron. This energy corresponds to a voltage of about one million. The remaining energy of the gamma photon should be divided equally or unequally between the two material particles produced. Hence the energy of a positron should in the best case always be one million volts less than the energy of the gamma photons, and this is actually found by experiment to be the case.

The converse process, the conversion of matter into radiation, is also possible according to Dirac's theory, for if an electron should fall back into a hole in the states of negative energy, it would exactly fill it. The electron would disappear, and so would the hole which, as we have said, is a positron. Radiation equivalent to the matter which disappears should then come into being. And certain experiments do indeed seem to show that gamma ray quanta may result from the mutual annihilation of an electron and a positron. The discovery of positrons has shown that we were wrong in assuming that positive electricity can only exist in combination with matter. In addition to electrons, which are the smallest known units of negative electricity, positrons exist which are the smallest known units of positive electricity. Hence we are now acquainted with the following elementary constituents of matter:

Alpha particles with mass 4 and charge $+2$
Protons with mass 1 and charge $+1$
Electrons with mass almost 0 and charge -1
Neutrons with mass 1 and charge 0
Positrons with mass almost 0 and charge -1

Dirac, in his Nobel prize oration, remarked that the number of kinds of elementary particles had in the last few years shown a disquieting tendency to increase. We must agree with him, when we consider the above table, and are inclined to ask whether we even now have discovered them all. The answer is quite uncertain. But suppose that we can assume the physicist to have at last all the parts in hand, let us hope that we do not then have to confess that the intellectual bond between them is missing. Let us seek for this.

The alpha particles are certainly complex. However, we can calculate, whether we reckon them to be made of protons and electrons, or of protons and neutrons, the mass defect, that is the loss of energy due to their combination; we find this in either case to be so great as to entirely explain the stability of their structure, in spite of its complexity. In order to disrupt the alpha particle into its parts, an equal amount of energy would be necessary. But it is certainly not a primary constituent of matter.

When neutrons and positrons were yet unknown to us, it was supposed that all nuclei were made up of protons and electrons (p. 25). This theory, which assumed the existence of electrons in the nucleus since they are emitted as beta rays, met with two principal difficulties. In the first place, electrons cannot, according to the quantum theory, be accommodated in the dimensions of the nucleus. They are the structural units of the shell of the atom, the properties of which can only be understood on these lines. They do not fit into the nucleus. The second great difficulty appears in the case of those radioactive elements which emit beta radiation. Measurements have shown that electrons are ejected from the individual atoms of any particular kind with very variable energy. Nevertheless, the residues of the atoms remaining after decay all possess the same energy. What have the slower electrons done with the energy which is wanting in them as compared with the fastest? It cannot be found experimentally. It would seem as if the law of conservation had been violated. Bohr quite seriously suggests that it might not hold in the place of nuclear transformation.

Perhaps this difficulty is only due to our assuming that

electrons exist in the nucleus. Heisenberg has proposed a new theory of atomic nuclei in which the electron does not appear as a constituent. He assumes that in the nucleus only neutrons and protons are present. If the number of the first is n_n, and that of the second n_p, then the number of the protons is equal to the nuclear charge. In addition, we then have a certain number n_n of neutrons in the nucleus. The atomic weight is given by the sum of the two, $n_n + n_p$. But how is such a nucleus held together? We no longer have the attraction of the negatively charged electron. Furthermore, how are we to explain the emission of beta particles by many radioactive elements, when these are certainly electrons, which are no longer present in the nucleus?

Let us first consider what are the forces acting between the components of the nucleus? The protons must repel one another on account of their positive electric charge. Let us call this force of repulsion K_p. What forces are we to assume to exist between a positively charged proton, and a neutron, and further between two neutrons? Heisenberg assumes that in the minute space of a nucleus, forces are active similar in character to those which bind together a neutral hydrogen atom and a positively charged hydrogen ion or two neutral hydrogen atoms (p. 187). In accordance with quantum chemistry (we cannot here explain more fully), the attractive force K_{np} between a neutron and a proton should be very much greater than that between the electrically uncharged neutrons K_n. These attractive forces, $K_{np} + K_n$ act in opposition to the repulsive force K_p, and must overcome it if the nucleus is to hold together. The greater the total force of attraction,

$$K = K_{np} + K_n - K_p$$

the more firmly is the nucleus held together, and the greater is its stability. We must also note that in the above expression, as we have just remarked, K_n is small compared with the other two forces.

In the case of elements which only contain a few protons, in other words those at the beginning of the periodic system (p. 16) K_p is also small, and hence we have approximately, $K = K_{np}$. If this force between the neutrons and protons is

to be large, many such pairs must exist; that is to say, the elements having few protons must have a number of neutrons equal to that of their protons. The atomic weight should then be about double the atomic number. And this is exactly what we find in the case of all elements up to calcium ($n_n=20$, $n_n+n_m=40$). When the number of protons becomes greater, the gradual increase in repulsion between the protons (K_p) must be compensated by the attraction of an increasing number of neutrons (K_n). The higher the atomic number of the elements the greater the excess of neutrons over protons, and the greater the excess of the atomic weight over twice the atomic number.

Heisenberg now puts this question: When is such a nucleus stable, and when can it fall apart when it is not stable? Let us first suppose that we have a nucleus consisting only of neutrons. Such a nucleus would be stable, for it would be held together by the force $K=K_n$ but its stability would be improved if one of the neutrons were replaced by a proton, for then the total force would be increased for the mutual attraction between a neutron and a proton is greater than that between two neutrons. Heisenberg assumes that a transformation of this kind in the nucleus comes about of itself, whereby the stability is increased, but this transformation of an electrically neutral neutron into a positively charged proton is only possible if at the same time an equally large negative charge escapes. Heisenberg assumes that just as an electron of the atom shell can pass from one possible quantum state to another more stable, with emission of a photon, hitherto undetectable, so in the atomic nucleus are the neutrons and protons only different quantum states. A neutron can change into a proton, and with the change, an electron comes into being and is emitted.

$$\text{Neutron} \rightarrow \text{Proton} + \text{Electron}.$$

This transformation of an atomic nucleus, rich in neutrons and poor in protons is thus perceived by the fact that it is a radioactive element which emits beta rays. The newly formed element may also again be one subject to beta ray transformation. The tendency of neutrons to be transformed into

protons will only cease when the number of neutrons as compared with the protons is no longer very great, that is, when the further change of a neutron into a proton leads in spite of the growing attractive force K_{np} to the continually growing repulsion of the protons K_p becoming too great. At this point, the atom is stable, at least as regards beta radiation. But if the number of protons is now too great as compared with that of the neutrons, the stability may be improved by the opposite transformation, which results in increasing the total force. A proton must now change into a neutron, and in this process a positive charge must be thrown out of the nucleus. We should expect a radioactive element which in decaying emits either a positron, a proton, or an alpha particle. Now protons are, as we know (p. 202), very tightly bound, but elements emitting alpha rays are found in considerable numbers in nature; and radioactive elements which decay with emission of positrons have also been discovered. We shall speak of them presently. From all this, we conclude that it is possible to assume that atomic nuclei are made up of protons and neutrons alone; this assumption is able to explain satisfactorily many experimental facts.

Let us now turn to the other problem of nuclear physics, the apparent violation of the conservation of energy when beta rays are emitted. The electrons forming these rays are emitted with very varying velocity, and nevertheless the new atoms which remain behind all possess the same energy. Pauli first suggested that there might be a kind of stowaway in the atom carrying the quantity of energy which is missing in those particles which are not emitted at maximum speed. These supposed particles, which have hitherto succeeded in remaining hidden, may owe their obscurity to the fact of their having a very small mass, perhaps even smaller than that of the electron, and also to their being electrically neutral. These particles which are supposed to be emitted along with the beta particles have been given the name "neutrino". On this supposition, the last equation would be more correctly written as follows:

$$\text{Neutron} \rightarrow \text{Proton} + \text{Electron} + \text{Neutrino}.$$

These neutrinos which are called in as a *deus ex machina* to

save a theory in danger of shipwreck, have not yet been discovered experimentally. The Italian physicist Fermi, well known by his statistical contribution to quantum physics, has attempted to work out the consequences of this idea. He succeeded in correctly calculating the dependence of the half periods (p. 194), of the various beta ray-emitting elements upon the velocity of the fastest electrons which they emit, and he also calculated the proportions in which rays of various energies are emitted. It also follows that the mass of the neutrino must be zero, or at any rate much smaller than that of the electron. However, a great deal more, perhaps of the most essential character, is required to clear up the position completely. "The development of the quantum theory up to date points to the fact that an understanding of those parts of atomic physics which are still unexplained can be only obtained by going still further than heretofore away from concreteness and objective comprehensibility" (Heisenberg). We have already mentioned that radioactive elements which decay with emission of positrons have now been found to exist. This interesting discovery was made by Curie and Joliot in 1934. They found that the known stable elements possess radioactive isotopes, which can be formed experimentally. When aluminium or magnesium is bombarded with alpha particles, a mixed radiation which can be observed in a cloud chamber is emitted; it consists of electrons, protons, and positrons. When the source of alpha rays is removed, the positron radiation continues for several minutes. The discoverers of this phenomenon ascribe the following equations to it.

$$\mathrm{Al}_{13}^{27} + \mathrm{He}_2^4 \rightarrow \mathrm{P}_{15}^{30} + \mathrm{Nn}_0^1; \quad \mathrm{P}_{15}^{30} \rightarrow \mathrm{Si}_{14}^{30} + e^+$$

The resulting "radiophosphorus" is thus an artificial radioactive isotope of ordinary phosphorus, and it decays with a half period of 3·25 minutes into a stable isotope of silicon with emission of positrons (e^+). The life of the isotope does not depend upon the energy of the radiation which produces it. Curie and Joliot prove their point by showing that the isotope has the chemical properties of phosphorus. They dissolve the aluminium which has been subjected to beta-ray action in hydrochloric acid. The result is the production of hydrogen

gas, and, if phosphorus is present, of phosphoretted hydrogen, and the hydrogen is actually found to be radioactive while the remaining solution is inactive. If the aluminium, on the other hand, is dissolved in aqua regia, the phosphorus should be oxidised, and should not escape. If zirconium is present in the solution as well, the result should be the formation of insoluble zirconium phosphate, and in this case the radioactivity is entirely confined to the precipitate.

Curie and Joliot, as well as other workers, have now produced a whole number of such artificial radioactive elements. In every case they emit positrons which, like the electrons emitted in normal radioactivity, have a range of energies. It has also been found possible to employ protons and deuterons in place of alpha particles to produce artificial radioactivity. The products sometimes emit positrons, sometimes negative electrons (beta rays, or "negatrons", as there is now a tendency to call them).

It is particularly interesting that Fermi and his collaborators have succeeded in using neutrons to transform the same light elements which had previously been transformed. More than this, they found that these electrically neutral projectiles are even able to disrupt many heavy nuclei which resist, on account of their powerful electrostatic repulsion, all other projectiles. Fermi has disrupted several dozen nuclei by means of neutrons. The following equations give two examples of such transformations.

$$P^{31}_{15} + Nn^{1}_{0} \rightarrow Al^{28}_{13} + He^{4}_{2}; \quad Al^{28}_{13} \rightarrow Si^{28}_{14} + e^{-}$$
$$Al^{27}_{13} + Nn^{1}_{0} \rightarrow Mg^{27}_{12} + H^{1}_{1}; \quad Mg^{27}_{12} \rightarrow Al^{27}_{13} + e^{-}$$

These two transformations are in both cases accompanied by the emission of electrons as in natural radioactivity. They have different half-periods, the first being about 2 mins. 17 secs., the second 12 minutes. The fact that in this case we get beta rays is easily understood. The absorption of a neutron results in a nucleus containing one neutron too many. This must then be transformed, with emission of an electron, into a proton. Fermi's discovery has been confirmed on all sides. It is possible that our ability to make artificial radioactive elements may prove to be of use in medicine, for natural preparations

are very expensive. The nuclei of heavy hydrogen supply us, as we have already said, with an especially good source of neutrons, and it is thus that we may hope to manufacture artificially radioactive elements.

In the course of his experiments Fermi was also able to decompose the element uranium, which possesses the highest order number, 92. A radioactive element is formed which, according to Fermi, behaves like the elements, manganese and rhenium, in the seventh column of the periodic system. This would point to its being a hitherto unknown element of atomic number 93 or 94.

CHAPTER IX

Science and Philosophy

Science aims at constructing a world which shall be symbolic of the world of commonplace experience. It is not at all necessary that every individual symbol should represent something in common experience, or even something explicable in terms of common experience.
A. S. EDDINGTON, The Nature of the Physical World.

One of the problems of the philosophy of nature, perhaps the fundamental problem, is this: How does it come about that we discover in nature *laws*, that is to say, unchanging relations between phenomena taking place in space and time? Are these laws characteristic of the external world, or do we ourselves put them into it; and if so, why are we successful in doing so? This fundamental problem already appears in Plato's Doctrine of Ideas, and in the mediaeval controversy concerning Universals. It is also the chief problem considered by Kant in his *Critique of Pure Reason*. Kant's answer has had most influence upon our own time; we are only able, he thinks, to acquire experience according to forms of thought which are fixed and innate in us, for example, causality and substance. We therefore put this order into nature. The philosophers who followed Kant, Schelling, Hegel, and their pupils, ended by believing that not only the forms of knowledge, but also its content, and hence the whole of reality, are present in ourselves, and that we should therefore be able to dispense with empirical investigation. The natural reaction against this "idealistic" philosophy was not slow to follow; the result was "positivism". The chief exponents of this philosophy were Auguste Comte, a philosopher, and Mach and Ostwald, both of them practical scientists.

Their fundamental thesis is as follows: Not only are our sense impressions the sole source of all our knowledge, they alone constitute nature. There is no difference between sub-

ject and object. Our sense impressions are both of these at once. The method of science is observation, and the refinement of observation is experiment. To this extent the positivists are also empiricists, like all scientific investigators. It is then our task to order our observations. Here the theorist goes to work. The task of theory is, as Kirchoff, the discoverer of spectrum analysis, once said, to "describe simply" and not to "explain"; for the latter must finally mean reference to something which lies behind the world of our senses. A close student of Mach has pointed out how strong are the bonds between his philosophy and that of the "Enlightenment" at the end of the eighteenth century. Both are "drunk with their own sobriety". A world lying behind our sense impressions cannot be tested by experience, and hence, according to the fundamental positivistic position, is physically inadmissible, a mere empty notion. Our sense impressions *are* the world, the only world about which we can learn anything by observation and experiment, the only world which exists for us. Metaphysics must be driven out of science. Only such questions are admitted as can be answered by observation. All really sensible questions are therefore susceptible of being clearly answered by science, and no "riddle of the universe" remains. Even the hypotheses which the physicist makes must not postulate anything which lies beyond the range of experience, for that would again be meaningless. All they can do is to predict new sense impressions. Their only value lies in "economy of thought". The atomic theory appeared to Mach to be a hypothesis of this kind, for it seemed impossible that we should ever be able to perceive atoms with our senses.

For the present, we will withhold our criticism of this view. The first important point is that this positivism, which was evolved in the last decades of the nineteenth century, is now being revived in the views of many present-day physicists. Heisenberg said: "Modern atomic physics does not deal with the nature and structure of atoms, but with the processes which we perceive when we observe atoms." Heisenberg's matrix mechanics was constructed precisely on these lines, and we have learned how successful it was. It is not based on

unobservable orbits and electron jumps in the atom, but upon observed frequencies and intensities of spectral lines.

The new positivism which is now coming into vogue also excludes all that is not measurable. Observation and experiment are the only sources of knowledge of the outside world. All statements that we make can only be statements concerning our experience. Nothing else whatever exists, and the "thing in itself" is rejected as a superfluous metaphysical speculation. Metaphysics is entirely useless to science; the scientist, since he is an empiricist, must logically also be a positivist.

As we know, the position and the velocity of an electron or proton cannot be measured accurately, for all means of measurement, such as instruments and light waves, since they themselves consist of electrons and photons, influence that which we are attempting to measure in a manner which we cannot control. Hence the "mass point" of classical mechanics, which at every moment can be given an exact value of position and velocity, is a meaningless notion, for only that which is measurable exists.

This also shows us plainly that Bohr's original theory is nonsense from the point of view of positivism. The Heisenberg uncertainty relation shows us that it attempts to tell us too much, for at any moment the electrons are supposed to have definite positions and velocities on definite circles and ellipses, and hence exactly defined values of energy, and that will not do. Both energy *and* time, position *and* velocity, can never be exactly measured at the same moment; hence, from the positivistic point of view, they do not exist.

We can draw a further conclusion. In classical physics, and also in Bohr's theory, the material point is supposed to describe the path that can be exactly followed, and we now conclude that such a thing as the path of the electron does not exist.

Let us consider this point a little more closely. The position of an electron can only be observed approximately. It can only be represented by a small circle, within which the electron must be (Fig. 58).

After lapse of a certain time, it would arrive, if we knew

its velocity exactly, within another equally large circle, the distance of which from the first circle can be calculated from the velocity. But since the speed of the electron is only known as regards magnitude and direction within certain limits, the new region in which we have to look for the electron is much larger. Our position is that of the sportsman who has just seen a hare run off in a certain direction. If he looks elsewhere for the moment, he is only able to tell approximately the new position in which the hare will be found; and if he does not look away, but keeps his eyes on the hare in order to follow it, an intelligent hare may perhaps perceive that it is under observation and may run in another direction; and an electron, although it does not take a different path on purpose, will, as we know, be disturbed by our observation, and will not therefore follow the same path as it would have followed if unobserved. The path of an electron is thus one of those concepts which cannot be measured experimentally, and hence, on positivistic principles, must be abandoned.

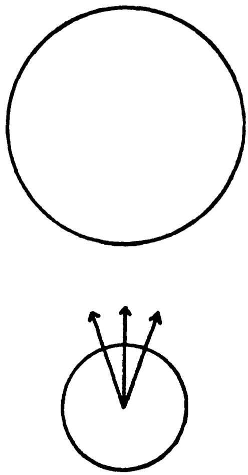


Fig. 58. The search for an electron, the position and velocity of which are only approximately known.

Thus the uncertainty principle leads to the important consequence that it is never possible to do more than determine average and statistical values for position and impulse, and never values for a single electron. The electron is only one member of a crowd, the behaviour of which in space and time is described statistically by a wave function. There is no other knowledge besides statistical knowledge. We must consider this result a little further, for a further important consequence for positivism flows from it. Classical physics had already made use of statistical methods; for example, the thermal properties of bodies could be calculated by statistical laws, which hold for the motion of many mass points. But classical physics nevertheless assumes that every mass point behaves

according to the strictly determined laws of Newtonian mechanics; the calculation could not, however, be carried out on these lines, for purely practical reasons, namely mathematical difficulties. In a similar way, the life insurance expert calculates the fate of a large number of insured persons, for he has no knowledge of the fate of a single individual, which may nevertheless be completely determined. This point of view we call *determinism.*

As we have already pointed out, classical physics is based entirely upon strict determinism. Mechanics, using the calculus developed by Newton for this purpose, calculates from the acting forces, and the positions and velocities of all masses existing at a given instant in planetary space, the motion of these masses for all past and all future time; the accuracy with which eclipses are predicted is the plainest proof of this method. Laplace's Demon, who was able to calculate the motion of all mass points in the whole universe, would be able to see that all that happens is completely determined. These ideas of Newton's are permeated with the spirit of the Baroque, and they continue to affect us even up to the present day. Goethe's "eternal iron laws" which rule the circle of all our existences, and Kant's awe when contemplating the starry heavens above us, are derived in part from Newton's mechanics. On the other hand, Newton's mechanics, taken as the programme of physics and even of the whole of natural science, resulted by reason of its determinism in the widespread view of nature as a huge machine; a machine which, once it has been wound up, runs down according to unalterable laws. This forms a basis for materialism of every kind, and for the unjustified and shortsighted conclusion that the human will is not free; indeed, that there is no such thing as spiritual existence.

The positivist, in view of the quantum theory, can no longer believe in the validity of determinism in physics, never hitherto seriously doubted. According to the uncertainty principle, it is only the average fate of many particles that is statistically determined; the behaviour of a single particle is completely undetermined. The causal principle as the physicist understands it (leaving aside the question whether philosophy

agrees) may be stated as follows: When the state of a "system", that is the position and velocity of all its parts, is known at a given moment, together with the forces which are operative in it and on it, its state at any other moment of time can be calculated. But if our system consists of mass points and photons, the causal principle is no longer a necessary form of thought for science. Heisenberg says: "If we state the law of causality in the form: 'If we know the present, we can calculate the future', it is not the conclusion but the premiss which is false, for we can never know the present completely in full detail", according to the uncertainty principle. Laplace's Demon could not even begin his calculation, since he could not know the initial position and velocity of even one single electron.

The only escape from the position is to suppose that the fate of a single particle is *actually* determined, but cannot be followed by us experimentally. Positivism denies us this escape as being fruitless and senseless, for it is impossible to state what the word "actually" means with reference to experiment. A well-known positivist, Arthur March,* expresses this view as follows: "Physics has only to deal with what it can actually observe. It may be true that the world 'in itself' is ruled by a hidden causality of the kind discussed, but this cannot interest the physicist, for the object of his investigation is not the world in itself, in which only the philosopher is at home, but the world as it appears to our senses."

Heisenberg also draws a similar conclusion: "Since all experiments are subject to the laws of quantum mechanics, the latter definitely proves the invalidity of the causal law." And Born says: "If it is impossible in principle to know all the conditions, that is causes, of a process, it is empty talk to say that every event has a cause."

This does not mean to say that only chance and irregularity rule the world, for that would make physics altogether impossible. Auguste Comte, whose ideas were derived from David Hume, regarded causality as a useful invention of the human mind. A profounder experience has now taught us

* *Die Grundlagen der Quantenmechanik.*

that it must be replaced by a law of a more general character, which allows us to predict from a state known to us with a certain degree of uncertainty, what will happen within certain limits in the future. The mathematical device used is, as we have seen, that of regarding this probability under the form of a wave. Only statistical predictions are possible, and they alone have any meaning. The philosopher Schlick says: "The causal principle is not a postulate in the sense in which this concept was used by earlier philosophers, for with them it was equivalent to a rule to which we must adhere under all circumstances; it is experience that decides regarding the principle; not with reference to its truth or falsehood—that would be meaningless—but to its utility." It is thus Hume and not Kant who is right as regards innate forms of thought. A form of thought may be shown to be, if not "wrong" or "empty", then "useless". Reichenbach says: "There are no principles *a priori* in the sense of being independent of experience. We only adhere to the most general principles of knowledge because they are found to hold good in experience." Since the positivist admits no world beyond our sense impressions and sensations, science for him consists, not in feeling one's way towards an objective world, or in the attempt to form a picture of such a world, but simply in the association of signs with our impressions. Philosophers are in danger of taking laws considered fundamental at a certain stage of physics as necessities of thought; in the opinion of positivists, they have given way to this danger. "The principle of causality is variable, and must submit to the requirements of physics." As our knowledge increases our philosophical attitude towards things ripens. Extreme positivists use very hard words concerning "the philosophy of the schools", which they regard as the "opium of science". They will have nothing to do with any philosophy which works above and beyond science, for they believe that it will in no way increase our knowledge of the world.

Let us now proceed to criticise positivism. Every scientist will always have a healthy mistrust for any mainly dialectical treatment of a problem in his science. While it is almost an occupational disease with him, as an empiricist, to over-

estimate empirical experience, this sceptical attitude towards all that is not experimentally proven is also his great source of strength. This sceptical position, which is most strongly represented in the positivist, is associated with the tendency to think on the following lines. Does not all debate concerning philosophical problems continually lead to the hopeless difficulty of ever arriving at generally recognised truth? Is it not better for the investigator to take as his motto, "This world is enough, what have I to do with eternity?"? And the positivists understand under the term "this world" the world as it presents itself to our senses. Thus Frank Thiess in his novel *The Centaur* describes positivism in the following passage: "The car obeyed its controls perfectly. The secret of the motor was only a riddle to the ignorant. Such a one might wrap it up in metaphysics. It was clear without being commonplace to one who knew. That which really exists does not need a metaphysical garment."

Let us consider both the sceptical position of the positivist, and the anti-metaphysical view. In the first place, we must agree that scepticism concerning all ideas unprovable by experiment has worked wonders for physics since the time when the wild imaginings of the Hegel school had to give way to empiricism. It got rid of many naïve ideas transplanted into the theory of new phenomena from our view of the everyday world, and it refused to allow many foolish questions even to be asked; it also abandoned many unnecessary concrete pictures. It called the speculative physicist at the right moment back from the clouds to earth, and gave him as well the impulse and tenacity to pursue the most unexpected and daring ideas whenever an experiment, otherwise inexplicable, could be explained by them.

But it also served as a drag upon the introduction of new hypotheses which were not immediately able to demonstrate their relation to the sensual world, as we see most clearly in the attitude of positivism towards the atomic theory. It opposed this theory, because it was no longer a working hypothesis for connecting sense impressions together, but an opinion concerning a reality beyond sense experience. Yet it was not positivism, but faith in the reality of atoms, even

though we could not perceive them directly, which led to the discovery of an abundance of new facts concerning the micro-world. By resolutely banning hypotheses which cannot at the moment be proved experimentally, science would be fettered, perhaps at a point when its greatest progress is possible.

Let us now turn to the anti-metaphysical view of positivism. It is a mistake on the part of the positivist to believe that a denial of the existence of the unobservable abolishes metaphysics. This very denial of a world beyond our sense impressions is itself a metaphysical statement; namely that sense impressions, or perhaps more generally, experience, is the only reality. But this is no necessary consequence of empiricism, and therefore to be accepted by every physicist; it is true that it is equally impossible to prove the statement to be untrue. It is rather a metaphysical hypothesis. It definitely follows David Hume, and denies the part ascribed by Kant to reason in experience. The "school" philosophers regard this sharp rejection of Kant's theory of knowledge as completely mistaken, and they too are not sparing with hard words.

"If we allow this point of view, when it is explained at length, really to sink into our minds we are almost staggered by the hopeless barrenness of this prospect, staggered at the idea that so many minds of scientific eminence are unable to look over the fences around their science, and by the fact that the greatest philosophers have lived in vain as far as they are concerned."* And Dingler says of positivism: "It shows strong affinity to the most primitive form of materialism." † He accuses the positivists of "a complete want of critical attitude towards their own mode of thought, which they regard as absolute". Among physicists, positivism has found its most relentless opponent in Max Planck. He has published many attacks upon it.‡ His chief reason is as follows: If experiences are really the absolute, then this can only refer to our own experience. This would mean that every physicist

* Al. Müller, *Die philosophischen Grenzfragen der modernen Physik.*

† *Geschichte der Naturphilosophie.*

‡ In English translation: *The Universe in the Light of Modern Physics* (1931), *Where is Science Going?* (1933).

must have his own physics, and it is quite incomprehensible why a universally recognised science exists. Positivism if thought out strictly to the end must lead to solipsism, and to a denial of the existence of science independent of the individual scientist, that is to say objective science.

Hence, the problem is to find another point of view as an alternative to the positivistic metaphysical view which fails. This view is, in Planck's opinion, the assertion of the existence of an external world. "The problem of physics is then altered. It does not consist in describing experience, but in understanding the real external world." From the correct assumption of positivism, that our sense impressions are the only means by which the external world acts upon us, Planck concludes that "the two statements: 'There exists a real external world, independent of ourselves', and 'The real external world cannot be known directly' form together the hub of the whole of modern physics". This is the source of the irrational, Faustian side of this science. Its goal lies beyond experience, and hence can never be attained completely, but only sought by endless roads which approach it as a curve approaches its asymptote. This represents fairly the view commonly held by those present-day physicists who are not positivists.

Their further epistemological views exhibit many varieties. There is agreement, not always plainly expressed, with Kant that experience is formed from sense impressions by the reason, which deals with them according to its innate "forms of perception" (as Kant terms them), space and time, and "forms of thought", the categories; the most important of the latter are substance and causality. Here the physicist feels it incumbent upon him to stress the fact that these forms are no more than forms without content, whereas Kant still makes them depend too narrowly upon a certain content, namely that of his own contemporary classical physics. The theory of relativity has taught us not to define too narrowly non-Euclidean geometry and the forms of thought, space and time, as was formerly done in the belief that only thus was Kant's point of view being upheld. In quantum physics, the fact must be stressed that the form of thought

causality certainly compels us to assume a strict law in the temporal succession of natural events, but not to assume particular forms of this succession. This form must rather be sought anew from time to time. By means of inductive investigation, that is by observations which are pointed by experiments and by logical mathematical treatment of them, we construct for ourselves a system of concepts and symbols. Planck calls it "the physical world picture". This gives us on the one hand as simple a picture as possible of the sensual world, and on the other hand as correct a picture as possible of the external world, which, in principle, cannot be known completely.

In this way, we are able to describe one aspect of the world, namely that which can be expressed in measurements and figures, as the English physicist and astronomer Eddington has pointed out.* All other aspects of the world lie outside the range of physical investigation: but they are no longer denied in the manner of the positivist. In this physical world picture, as Planck points out, the anthropomorphic elements, namely those brought in by human sense organs and the measuring instruments which increase their acuteness, are driven out again to a steadily increasing degree. Our continually increasing power to affect the course of nature by making use of this world picture, as is seen in the progress of physical technology, is for the physicist a guarantee of the effectiveness of his method, of the fact that his picture represents the true features of the external world with ever increasing accuracy. This must be stressed as against doubts put forward by other philosophers.

In this view, every new discovery represents a deeper insight into reality, as opposed to the positivistic belief to the contrary. It represents an improvement in the concepts and hypotheses which we have formed, and which make up this world picture, and it is imagination, without which no scientific research is possible, which guides the investigator in his search for better symbols. The question whether these have been chosen rightly or wrongly is answered by experiment, which decides the point. And it is also a matter for

* *The Nature of the Physical World.*

our progressing experience to decide what is to be regarded as unchangeable, as substance in the metaphysical sense. The view, considered by Kant as self-evident, that substance is identical with "mass", has long ago been abandoned. Mass is not something unchangeable. In the same way, the content of the principle of causality can only be decided empirically.

The question which concerns us in this chapter is the contributions made by quantum physics to our world picture. We have seen that the classical world picture no longer suffices, but that, on the contrary, it must be fundamentally revised. Which of its fundamental concepts can we make use of? We have already learned the answer given by the positivist to this question, and we further discussed the criticism of positivism by metaphysicians, as we will shortly term them. We will now ask what answer the latter can give to our question.

The first question is the concept of the "material point" which has now been found so full of contradictions. It is clear that "the previous central meaning of this concept must be finally sacrificed" (Planck). The properties which we ascribe to it in the macroscopic world directly accessible to our senses do not really belong to it. It has not a definite velocity at a definite position, nor a definite energy at a definite time. These notions are purely ideal; they can be assumed as real in the macro-world only on account of the minuteness of Planck's constant *h*, and its position in the uncertainty relationships. But we must not believe that the concepts which hold for the world of large objects have absolute validity. Since corpuscles possessing exactly definite values of position, velocity, and path, and likewise waves, were useful concepts in macro-physics, we formerly thought that something of the kind must of course also exist in the micro-world. Kant postulates the category of "substance which remains unchanged amid the multiplicity of phenomena". All classical physicists, including Kant, regarded the material points of mechanics as such substances. They are not so. On the contrary, they are particles and waves at the same time: Eddington calls them "wavicles". They are at once the principle of eternal being of the Eleatic philosophy, and also

that of eternal becoming of Heraclitus. Are wavicles still substances in Kant's sense, or must we extend our epistemological frames? Are the forms of thought held to be necessary by Kant perhaps still too closely defined as to content, for that which has real physical existence, as we already know (see p. 69), is the quantum of action? But this belongs to the four-dimensional, space-time world, which cannot be directly perceived. We no more possess sense organs capable of perceiving this world, that the blind man can perceive the world of colours.

Reality, which includes both macro- and micro-worlds, cannot be described unambiguously by means of our previous concepts of the macro-world. If we make the attempt, we get into difficulties, for both electrons and photons exhibit characteristics both of the wave and of the particle. There is nothing of this kind in the macro-world, and hence we cannot completely describe matter and light by any concept derived from it.

For they contradict one another when we assume them without consideration. They supplement one another, and can be converted one into the other, by the well known Planck-de Broglie relationships. There is no point in enquiring as to the mechanism which lies behind this possibility of transformation. That would mean seeking something in the macro-world to form a comparison. No such thing exists. We can find nothing at all in the whole range of the world accessible to our unaided senses, which has at once the properties of a wave and a particle, as has an electron or a proton. Every atomistic process is on the contrary, as Bohr says, "an individual process which cannot be further described", the transition from one quantum state to another.

We are unwilling to do without the notions of the macro-world, even when they are only of limited validity in the atomic world. For they have formed our language in the course of thousands of years since the earliest times of human civilisation. As we learn our language, we absorb these experiences of our distant ancestors, and both their true and their misleading ideas. Language saves us an enormous amount of work, but it also hinders us when, as to-day, we are

attempting to find new concepts to formulate a much richer experience of nature. Until a short time ago, the concepts formed by language also sufficed to describe reality in a non-contradictory manner. This is no longer true of our statistical point of view constructed by Bohr, Heisenberg and Born. Bohr says: "We have no resource but to express ourselves by means of a word painting." We are confronted with "difficulties which arise from the fact that all ordinary words of our language have been coined by our accustomed points of view, from which the existence of a quantum of action is irrational. As a result of this situation, even such words as 'be' and 'know' lose their unambiguous meaning". Dirac is of the opinion that in recent times it has become more and more evident that nature works according to quite a different plan from that on which our ordinary views are based.

The laws of nature do not relate directly to a world which we are able to imagine in space and time, but to a something of which we can form no concrete picture. The new theories are founded on physical concepts which, if we set aside our mathematical apparatus, cannot be described by means of expressions familiar to students in the past. Indeed, they cannot be adequately explained at all in words. Just as everyone after his arrival in the world must gradually acquire fundamental concepts such as nearness, identity, etc., so also can the new concepts of physics only be acquired by a gradual familiarity brought about by frequent use of them and their properties. Statement in words becomes a mere matter of words and no more, as regards a profounder view. We coin such new words as "wavicle".

The mathematical formalism which has been developed allows us to understand more completely than ever the known physical world which to-day is richer in content than ever before. Do we really *understand* the world better than we did without the aid of physics? "In physics, we understand the formal behaviour of an unknown quantity. This results from our advance along the road taken by physics since Democritus. Just as the atoms of Democritus and the founders of chemistry possessed neither odour nor colour, so also, do we think to-day,

are they devoid of any geometrical properties" (Heisenberg: *Zur Geschichte der physikalischen Naturerklärung*).

The realm of mathematical symbols with which the physicist strives to grasp the world to-day cannot be entered in this book; we will rest content with having rendered comprehensible a few of the leading ideas. If we follow them up, we are as much captivated by the beauty of the formulation, and the depth of the connections, as by a great work of art.

In this stage of science, the mathematical symbol, such as the wave equation of Schrödinger, ceases to be a tool and becomes an end in itself. These waves are almost more than form. They are a form to which we can ascribe no substantial content, or at least only one of a psychical nature. For we cannot find this something that undulates. We can only interpret them as waves of probability. The fact that Schrödinger's equation has quite a special mathematical form, the same as that which describes the wave processes of classical physics, is the sole link between quantum processes and classical waves. This is the reason why we are able to interpret natural phenomena up to a certain point in terms of waves; but the waves are only a picture. What really exists is the mathematical foundation.

This greatly increased importance of mathematics in physics was already evident in the theory of relativity, and it has led some English astronomers, such as Eddington and Jeans, to make confessions of faith of an almost religious character and of a very personal kind. A new spiritualism is in process of formation. The world of classical physics, constructed of mass points, seems like a large machine. The world of quantum physics, constructed of waves of light and matter, appears to be only formally determined by these waves. If nothing material can be found underlying it, the alternative is something of a psychical or spiritual character.

We are familiar with the relationship of the poet Goethe to nature. He feels himself perfectly at rest in it: "Nature brought me into the world, and will also lead me out of it. I trust her. Let her do with me what she pleases. She will not hate her own work. I did not speak of her. No, what was true and what was false, all was spoken by her. Hers alone is the

guilt, hers alone is the reward." The astronomer Jeans, looking out into the distant world of space, shudders as he thinks of the terrible loneliness and isolation of mankind in the world.

But the great astronomer knows more than this, and though not possessing the warmth of Goethe's feeling for nature, he is not overwhelmed with loneliness. The discovery that something mathematical is the last reality is a consolation to him, and a proof that he is not alone and forsaken in a frigidly indifferent world; for he himself is able to follow in thought this deepest reality of the world. Here we have a revival of the most ancient ideas of the Pythagoreans.

Bavink, in his little volume *Science and God*, also stresses the importance for the general spiritual attitude of mankind of the transformation which has taken place in our physical world picture. As long as the scientist believed that the mechanistic world picture of classical physics could be carried through consistently, and while this view was leading to most impressive successes in physics, and to increase of our power over nature, the influence of mechanistic thought extended to the whole of cultural life. Bavink goes on to point out that in place of social and cultural structures formed by connections which had grown organically, there arose more and more artificially constructed arrangements with purely utilitarian aims. Life was often regarded as nothing more than a pattern of atoms. But nowadays, we see that physics increasingly rejects a purely mechanistic explanation, and that the change in this direction began already in the classical period.

In this way, the new physics offers a contribution to the old metaphysical problem of the relationship between body and soul.

Mass points as the substance of the physical world have disappeared, and have handed over their role to something which is at once wave and corpuscle, and can be well described by Schrödinger's wave equation. This does not mean that we agree with the positivist in denying the existence of material points, but that we find, in Kant's language, a new content for the form of thought "substance", a content which

better describes our sense impressions, and reproduces symbolically a richer region of the unknowable world. This is a new step on the road which is always taken in the increase of physical knowledge. Hence the physicist who is a metaphysician must regard the mathematical description of the world as revealed to our senses as the sole problem of the investigator. It is not his task to "explain" this world in terms of something lying behind all experience; indeed, he will always be a pessimist as regards the possibility of understanding the inward nature of the world. But as a scientist, he is an optimist. For here his goal is more modest. It is simply to find symbols, increasingly more adequate, to form a unified description of the world of experience. These symbols can be tested by their power to predict new experimental facts.

As regards the mass points which we may retain for want of a better concept, we are only able to make statistical statements concerning their distribution in space and time, using for the purpose the picture of a wave. The individual fate of these material points remains quite undetermined. This renunciation of detailed knowledge is no disadvantage for physics. On the contrary, the new concept of wavicle is much more useful than the old Newtonian mass point, for it includes both micro- and macro-mechanics, since the latter is contained as a limiting case in the quantum theory. The wave equation describes the average behaviour of a large number of wavicles, and that too with the complete accuracy proper to such an equation. Without saying anything concerning the behaviour of individual wavicles, it states everything that physics has hitherto been able to tell us concerning the whole of matter, whether in large or small dimensions.

We have seen how the concept of the mass point must be revised. The philosophical foundation differs in the case of positivists and metaphysicians, but the physical consequence is the same; the mass point must be replaced by the statistical wave.

We now turn to another debatable question, namely the content which is to be ascribed to the causal principle. While the standpoint of the positivists is agreeably clear and definite,

guilt, hers alone is the reward." The astronomer Jeans, looking out into the distant world of space, shudders as he thinks of the terrible loneliness and isolation of mankind in the world.

But the great astronomer knows more than this, and though not possessing the warmth of Goethe's feeling for nature, he is not overwhelmed with loneliness. The discovery that something mathematical is the last reality is a consolation to him, and a proof that he is not alone and forsaken in a frigidly indifferent world; for he himself is able to follow in thought this deepest reality of the world. Here we have a revival of the most ancient ideas of the Pythagoreans.

Bavink, in his little volume *Science and God*, also stresses the importance for the general spiritual attitude of mankind of the transformation which has taken place in our physical world picture. As long as the scientist believed that the mechanistic world picture of classical physics could be carried through consistently, and while this view was leading to most impressive successes in physics, and to increase of our power over nature, the influence of mechanistic thought extended to the whole of cultural life. Bavink goes on to point out that in place of social and cultural structures formed by connections which had grown organically, there arose more and more artificially constructed arrangements with purely utilitarian aims. Life was often regarded as nothing more than a pattern of atoms. But nowadays, we see that physics increasingly rejects a purely mechanistic explanation, and that the change in this direction began already in the classical period.

In this way, the new physics offers a contribution to the old metaphysical problem of the relationship between body and soul.

Mass points as the substance of the physical world have disappeared, and have handed over their role to something which is at once wave and corpuscle, and can be well described by Schrödinger's wave equation. This does not mean that we agree with the positivist in denying the existence of material points, but that we find, in Kant's language, a new content for the form of thought "substance", a content which

better describes our sense impressions, and reproduces symbolically a richer region of the unknowable world. This is a new step on the road which is always taken in the increase of physical knowledge. Hence the physicist who is a metaphysician must regard the mathematical description of the world as revealed to our senses as the sole problem of the investigator. It is not his task to "explain" this world in terms of something lying behind all experience; indeed, he will always be a pessimist as regards the possibility of understanding the inward nature of the world. But as a scientist, he is an optimist. For here his goal is more modest. It is simply to find symbols, increasingly more adequate, to form a unified description of the world of experience. These symbols can be tested by their power to predict new experimental facts.

As regards the mass points which we may retain for want of a better concept, we are only able to make statistical statements concerning their distribution in space and time, using for the purpose the picture of a wave. The individual fate of these material points remains quite undetermined. This renunciation of detailed knowledge is no disadvantage for physics. On the contrary, the new concept of wavicle is much more useful than the old Newtonian mass point, for it includes both micro- and macro-mechanics, since the latter is contained as a limiting case in the quantum theory. The wave equation describes the average behaviour of a large number of wavicles, and that too with the complete accuracy proper to such an equation. Without saying anything concerning the behaviour of individual wavicles, it states everything that physics has hitherto been able to tell us concerning the whole of matter, whether in large or small dimensions.

We have seen how the concept of the mass point must be revised. The philosophical foundation differs in the case of positivists and metaphysicians, but the physical consequence is the same; the mass point must be replaced by the statistical wave.

We now turn to another debatable question, namely the content which is to be ascribed to the causal principle. While the standpoint of the positivists is agreeably clear and definite,

that of the metaphysician who rejects the positivist position is much more difficult.

Kant formulated the causal principle in the following words: "Everything that happens presupposes something from which it follows according to a rule." In the first place, this does not say that only the fate of mass points can form a causal chain of events, although in Kant's time the physics of that day gave this interpretation as the most obvious. Secondly, the wording is such that the nature of the connection may be regarded as looser than is commonly assumed. The causal law is certainly regarded by Kant as a necessary presumption on which to base research, but the presumption is only of a formal character. It is the business of science to give it content. So much is certain: the classical Demon of Laplace, who was able to calculate strictly the fate of every individual mass point, receives his dismissal from the metaphysician as well as from the positivist. For he cannot even begin his task, since the initial data are always uncertain. The determination of individual mass points as known to classical physics thus disappears on account of the uncertainty principle. But, as Planck points out, the position is changed when we regard the waves of matter as the symbols which form the fundamental concept of the physical world picture. They can be described by a classical wave equation, and hence follow strict causality according to the forms of classical physics. Only, their relationship to the world of our senses is more complicated.

Nevertheless, an objection is raised by those who deny the validity of the causal principle. The strict course of natural law still does not appear to be guaranteed, for the waves only rule the *probability* of natural events. Single events do not take place with strict causality, but only with statistical regularity; hence the position has not been cleared up entirely. "It may possibly be true, that even the most complete observation of the total state of the world at a given moment would not allow the future course of events to be predicted with accuracy and certainty" (Schrödinger).

There remains a residue of doubt. One feels compelled to introduce into the necessity which rules the world of ex-

perience an element of chance. However, a reason can be given for the unsatisfactory character of this view. This is to be sought in the fact, not hitherto taken into account, that at every new observation an uncontrollable influence is exerted upon the object by the means of observation, as the uncertainty principle tells us. There are no events which are independent of all observations. "The present situation in physics", says Bohr, "has reminded us emphatically of the old truth that we are both spectators and players in the great drama of existence." We cannot in any way investigate physical objects without means of measurement, for example without our sense organs, and without appliances, such as light. They have no separate individual existence; they constitute reality only when combined with the means of observation. The line of division which we draw between the object and the subject is actually arbitrary within certain limits.

If we were able sharply to separate off reality, as an "ideal mind" might do, and as classical physics regarded as possible for everyone, it might be that a strict rule of law in nature could be shown to exist. This belief is a way out of the difficulty for the metaphysician; it is impossible for the positivist, since an "ideal mind" is for him a physically "empty notion".

From all that we have said, it is an open question for the physicist whether we must regard events as determined, or as undetermined within the limits set by the uncertainty principle. If we hope to have the question answered for us by further physical experiment, we are breaking with Kant's theory of knowledge, for not only the content of the category causality is thrown open to doubt, but the applicability of this form of thought itself. Experience is then to decide whether this form of thought is useful, or should be replaced by another, whereas according to Kant the categories are the means by which *all* experience is rendered possible, and hence they themselves are not susceptible of experimental test. Nevertheless, there are many who take up this purely empirical standpoint; von Laue however points out that the abandonment of a principle regarded philosophically as a

necessity of thought need only be provisional. Newton himself abandoned the postulate, in his time regarded as a necessity of thought, that all forces act only from point to point; in his law of gravitation, he introduced provisionally action at a distance from star to star through empty space. But in course of time, in this case only after the collection of a very great number of experimental data, gravitation was, as we have seen, explained by the general theory of relativity in quite a different way, and the old rejection of action at a distance was revived. Von Laue thus regards our renunciation of a causal interpretation merely as a working hypothesis, "which may be necessary as long as we continue to make use of the conceptual constructions of classical mechanics". This is a possible and perhaps necessary course to take. The actual solution of the present philosophical problem is thus postponed. The possibility always remains that a better statement of the problem would lead to a full and complete causal understanding of physical processes.

How difficult it is to state questions correctly is shown by a study of every separate science, and exemplified in our study of the physical ideas of the generation just past. How much more difficult may it not be to state the problem of causality correctly? Perhaps, as we saw in considering the concept of substance, the separation between the formal statement and the content of the causality principle is not yet perfectly satisfactory, as the positivists maintain, although they go too far. It may be that the great mathematician Hilbert, the founder of modern research into the axioms of geometry, is right when he says, "The *a priori* is no more and no less than a fundamental attitude, the expression of certain inescapable preliminary conditions of thinking and experiencing. But the limit between what we possess *a priori* and what we must acquire by experience, must be set otherwise than Kant set it; Kant greatly overestimated the part played by the *a priori*, and its range."

Let us return to physics. We have recognised that well founded concepts of classical mechanics cannot be upheld in the face of our newly acquired experience, and we must confess that improved concepts have not yet been found.

Between the concepts which have been taken from macrophysics into atomic physics, there exists, as Bohr puts it, a complementarity of the mode of description. Either we follow a particle exactly in space and time, when its energy and impulse are only inexactly determinable (as the uncertainty principle tells us), and can no longer be described in terms of strict causality, or we operate with strictly causal energy and impulse, whereupon the arrangement in space and time becomes completely indeterminate. We must not take the uncertainty principle to mean that we are prevented from penetrating into the secrets of nature. All that it tells us is, that a foolish question cannot expect a sensible answer. The world is more mysterious than we imagined in the days of classical physics. Our macroscopic mechanical concepts do not suffice to understand it. The uncertainty principle is not the expression of our ignorance of the moment; nature is indeed so constructed. Light and matter have this quite unpicturable double nature. Space and time, that is to say wave concepts, and energy and impulse, that is corpuscular concepts, supplement one another, and can only be applied together within certain limits. Light and matter do not *possess* definite positions and impulses, but an experiment made with them allows us to apply both concepts to explain our results within certain limits. These concepts do not really exist, but belong to our physical world picture, and are useful within certain limits.

This also answers the question as to what our experimental results in quantum physics teach us from the positivistic point of view regarding our world picture. The uncertainty principle forbids us to form a concrete interpretation of the whole of physics, namely the macro- and micro-worlds, by means of the classical concept of the mass point. If we nevertheless make use of it, and attempt to work with concrete pictures, instead of with Heisenberg's abstract matrix mechanics, that is to say, if we use the notions of a corpuscle and a wave derived from macro-physics, we are using inadequate pictures which contradict one another to a certain extent. The causality principle also fails for these inadequate concepts; even the most orthodox disciple of Kant will allow the physicist so

much latitude. The fate of individual corpuscles is undetermined, but their average behaviour is strictly determined by the probability waves. These inconvenient and more abstract ideas are obviously not to be regarded as final; but they must be nearer to reality, since they include a larger range of facts, namely those of both classical and quantum physics.

They are of course very difficult to conceive concretely. That cannot be otherwise, for only concepts derived from the everyday world are concrete and picturable, but as we use these notions more and more frequently they become more familiar. This has often happened in physics, as for example in the case of Newton's force at a distance, Faraday's conception of the field of force, and the space-time concepts of the theory of relativity. Heisenberg, indeed, will not allow that quantum physics is unpicturable. He asks what we mean by this term, and says that we regard a theory as picturable "when we are able to think out qualitatively in all simple cases the experimental consequences of the theory". Nevertheless most of us will find it more difficult to picture probability waves than electromagnetic waves, not to say the elastic waves of early optics. But the road of physical investigation leads away from the ideas which we form from our direct sense impressions to ideas arising from further experimental investigation and discussion of its results; these later ideas are more and more unlike our original notions. The difference from those we picture on the basis of our direct impressions of the world, and use in the beginning of physical investigation, becomes ever greater and greater. But the things themselves are not identical with our notions of them. The growing number of sense impressions, including the ever increasing number of experiments with the atomic world, leads step by step to a conceptual world, which is increasingly strange to the uninstructed. In succession, the notion of atoms was introduced into our conception of matter, then the structure of atoms was conceived in terms of moving electrons, then we passed on to matter waves conceived as electrical charge-density, and finally to probability waves.

It is noteworthy that in spite of the very different epistemological positions of the positivist and the metaphysician,

they agree perfectly as regards the conclusions to be drawn in respect of physics. This is due to the fact that physics as a branch of science is to a great extent independent of its philosophical foundations. For physics investigates the world of appearances, and only then can the philosopher enquire as to the meaning of this world.

What we have learned allows us to cast a glance beyond physics over other branches of scientific investigation. Bohr believes himself able to draw a conclusion important for biology. He regards it as impossible to comprehend the phenomena of life by means of the concepts of classical physics. These were adequate to describe the functioning of a macroscopic "dead machine", but they are not adequate to describe atomic processes. It was a mistake to believe that the object observed and the observing subject could be completely separated, and when we come to observing completely an organism, this separation is completely unthinkable. Anything living is taken apart and killed by complete observation. The causal and spatio-temporal view point of classical physics, and the purposive point of view of biology, appear to him as extremes. Between the two we have quantum mechanics, which proceeds statistically.

From this point of view, Bohr thinks that we may find new light thrown upon psychology. Just as we disturb atoms in their behaviour by observing them, so do we disturb our own psychical processes by introspection. Causality and free will thus appear as the idealised frames of physical and psychobiological law. But both of them are inadequate to solve the problem of body and soul, since they perhaps also stand to one another in a relationship of complementarity.

Heisenberg stresses the same point. Since the concepts of classical physics already fail to deal with events in the interior of the atom, how much less adequate must they be to deal with scientific regions lying still further from them! The principles of classical physics do not suffice for an understanding of the world as a whole. This does not mean to say that we have reached the limits of rational thought, but only that it is not permissible to apply the forms of thought of a particular period of physics to the whole of reality.

There are still plenty of riddles and problems left for physics to solve, but to-day we know a good deal more of fundamental importance than did the previous generation. One of the most important points into which insight has been gained is the following: the things of this world, radiation and matter, are not bodies, and not waves. Body and wave are like two different languages. They are fitted to exhibit a certain side of reality, but by means of them we can only grasp it in a veiled form. As the form is changed, the content becomes a little clearer. A "suit of clothes", to use Carlyle's expression, was laid aside with classical physics, since it no longer fitted. Through the deceptions of appearance, we have come a step nearer to the truth of Being. This knowledge will remain with us, and its best truths may be perhaps harvested when much that has greatly occupied us is left behind as non-existent appearance.

Scientific theories are like great breakwaters built out into the sea. The old breakwater was destroyed in the last great storm, but the new one, built on better foundations and reaching farther out, will resist better. But not for ever. Every scientific theory passes. Only nature itself is eternal. Even this temporal attribute is not just, for, like the concepts of body and wave, the category of time is a human measuring-rod which is laid upon things, and differs from them fundamentally. Nature is in all respects limitless, the Apeiron, as the genius of the old philosopher Anaximander of Miletus told him: "The origin of things is in the limitless. Whence they arise thither they also of necessity return." All investigation, indeed all limitation and separation of something single and concrete from the limitless, carries in itself its own destruction. Every scientific theory must some day pass away inevitably. Thereby it pays the penalty for having appeared out of the void. He who eats of the tree of knowledge must leave the paradise of the infinite, and must till the fields anew each year.

Just as every spring brings forth new creatures, so does the ever youthful spirit of Western science bring forth new flowers. The search for truth is like a climb in the high mountains. The air which we breathe becomes thinner and

clearer. The rich fertility of the valley gives place, more and more, to severe magnificence. The measures of things known to us in the valley become less and less applicable. We make use of others. We reach points from which our range of vision becomes much greater, while our survey of the familiar valley becomes clearer along with the view we gain of the new high world. We climb one peak and make a short rest, but already the next and higher one awaits us.

Index

Note. When an author's name alone is given, the reference is to some general view expressed by him

Index

Index

Index